8.66% APR is based on a Prime Rate of 8.25% as of August 1, 2006.

CHAPTER

2 First-Degree Equations and Inequalities

Earl invests a total of \$10,000. He invests part in Collins Feline Fanciers that pays a 9% dividend per year and the rest in Grutz Shipyards that pays an 8% dividend per year. If Earl receives \$870 per year from his investments, how much did he invest with each company?

2–1 ■ Solving equations

Terminology

An **equation** is a statement of equality. If two expressions represent the same number, we place an equality sign, =, between them to form an equation. We use the following example to show the parts of an equation.

$$\underbrace{2x + 9}_{\text{Left member of the equation}} \quad \underset{\substack{\uparrow \\ \text{Equality sign}}}{=} \quad \underbrace{7 - 4x}_{\text{Right member of the equation}}$$

A **mathematical statement** is a mathematical sentence that can be labeled true or false. $2 + 5 = 7$ is a true statement, and $3 + 4 = 8$ is a false statement. An equation that is true for some values of the variable and false for other values of the variable is called a **conditional equation.** The equation $x + 1 = 8$ is a conditional equation since it is true only when $x = 7$ and false otherwise.

A replacement value for the variable that forms a true statement is called a **root,** or a **solution,** of the equation. We say that a solution of the given equation *satisfies* that equation. The set of all those values for the variable that causes the equation to be a true statement is called the **solution set** of the equation. In the equation $x + 1 = 8$, the solution set is $\{7\}$.

If the equation is true for every permissible value of the variable, it is called an **identical equation,** or **identity.** For example,

$$4(a - 2) = 4a - 8$$

is true for any real number replacement for a and is thus called an identity.

Properties such as

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c \quad \text{and} \quad a + b = b + a$$

are further examples of identities. We will be concerned only with conditional equations in this chapter.

In this chapter, we are concerned with **first-degree conditional equations,** also called **linear equations.** In a first-degree conditional equation in one variable, the exponent of the unknown is 1 and the solution set will contain at most one root.

If we wish to solve an equation such as

$$5(2x - 1) = 7x + 10$$

we go through a series of steps whereby we form equations that are **equivalent,** *having identical solution sets,* to the original equation. *Our goal is to form equivalent equations until we isolate the unknown in one member of the equation and our equation is in the form* x = n, where n is some real number. The following are equivalent equations whose solution set is $\{5\}$.*

$$\begin{aligned} 5(2x - 1) &= 7x + 10 \\ 10x - 5 &= 7x + 10 \\ 3x - 5 &= 10 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

Since an equation is a statement of equality between the two members of the equation, identical quantities added to or subtracted from each member will produce an equivalent equation. This is called the **addition property of equality.**

Addition property of equality

For any algebraic expressions A, B, and C, if $A = B$, then

$$A + C = B + C$$

Concept

The same expression can be added to each member of an equation and the result will be an equivalent equation.

In chapter 1, we defined subtraction in terms of addition, therefore *we can use the addition property of equality to* **subtract** *the same expression from both members of an equation.*

■ Example 2–1 A

Find the solution set.

1.

$4x - 2 = 3x + 7$	
$4x - 3x - 2 = 3x - 3x + 7$	Subtract 3x from both members
$x - 2 = 7$	
$x - 2 + 2 = 7 + 2$	Add 2 to both members
$x = 9$	Solution

The solution set is $\{9\}$.

*The formation of these equivalent equations is achieved by using the addition and multiplication properties of equality.

2.
$$
\begin{aligned}
2(3x-1) &= 5x+2x-3 \\
6x-2 &= 7x-3 && \text{Simplify} \\
6x-6x-2 &= 7x-6x-3 && \text{Subtract } 6x \text{ from both members} \\
-2 &= x-3 \\
-2+3 &= x-3+3 && \text{Add 3 to both members} \\
1 &= x && \text{Solution}
\end{aligned}
$$

The solution set is $\{1\}$.

Multiplication property of equality

The addition property of equality along with the properties of real numbers is sufficient to solve many of the equations that we encounter. However, they are not sufficient to solve equations such as

$$4x = 12 \quad \text{or} \quad \frac{3}{4}x = 15$$

Recall that we want our equation to be of the form $x = n$. This means that the coefficient of x must be 1. To achieve this, we need the **multiplication property of equality.**

Multiplication property of equality

For any algebraic expressions A, B, and C, where $C \neq 0$, if $A = B$, then

$$A \cdot C = B \cdot C$$

Concept

An equivalent equation is obtained when we multiply both members of an equation by the same nonzero expression.

In chapter 1, we defined division in terms of multiplication, therefore *we can use the multiplication property of equality to* **divide** *both members of an equation by the same nonzero expression.*

Example 2–1 B

Find the solution set.

1.
$$
\begin{aligned}
4x &= 12 \\
\frac{4x}{4} &= \frac{12}{4} && \text{Divide both members by 4} \\
x &= 3 && \text{Solution}
\end{aligned}
$$

The solution set is $\{3\}$.

Note We could have multiplied by the reciprocal of 4 to solve the equation. That is,

$$
\begin{aligned}
4x &= 12 \\
\frac{1}{4} \cdot 4x &= \frac{1}{4} \cdot 12 && \text{Multiply both members by } \frac{1}{4} \\
x &= 3
\end{aligned}
$$

We should be familiar with the idea that to divide by a number is the same as to multiply by the reciprocal of that number.

2. $$\frac{3}{4}x = 15$$
$$\frac{4}{3}\cdot\frac{3}{4}x = \frac{4}{3}\cdot 15 \qquad \text{Multiply both members by the reciprocal of the coefficient of } x$$
$$x = 20 \qquad \text{Solution}$$

The solution set is $\{20\}$.

Recall that when we divide by a fraction, we invert and multiply. Therefore if the coefficient of the unknown is a fraction, we will multiply both members of the equation by the reciprocal of the coefficient.

3. $$2.6x = 10.4$$
$$\frac{2.6x}{2.6} = \frac{10.4}{2.6} \qquad \text{Divide both members by 2.6}$$
$$x = 4 \qquad \text{Solution}$$

The solution set is $\{4\}$.

Using the given theorems and the properties of real numbers, there are four basic steps for solving a linear equation. We shall now apply these to the equation

$$5(2x - 1) = 7x + 10$$

Solving a linear equation

Step 1 *Simplify the equation.* Perform all indicated addition, subtraction, multiplication, and division. Remove all grouping symbols. In our example, step 1 would be to carry out the indicated multiplication in the left member.

$$5(2x - 1) = 7x + 10$$
$$10x - 5 = 7x + 10$$

Step 2 *Use the addition property of equality to form an equivalent equation where all the terms involving the unknown are in one member of the equation.* By subtracting $7x$ from *both* members of the equation, we have

$$10x - 5 = 7x + 10$$
$$10x - 7x - 5 = 7x - 7x + 10$$
$$3x - 5 = 10$$

Step 3 *Use the addition property of equality to form an equivalent equation where all the terms not involving the unknown are in the other member of the equation.* Adding 5 to *both* members of the equation, we have

$$3x - 5 = 10$$
$$3x - 5 + 5 = 10 + 5$$
$$3x = 15$$

Step 4 *Use the multiplication property of equality to form an equivalent equation where the coefficient of the unknown is 1.* That is, $x = n$. Dividing *both* members of the equation by 3, we have

$$3x = 15$$
$$\frac{3x}{3} = \frac{15}{3}$$
$$x = 5$$

The solution set is $\{5\}$.

To check our solution, we substitute the solution in place of the unknown in the original equation. If we get a true statement, we say the solution "satisfies" the equation.

In our example, $5(2x - 1) = 7x + 10$, we found that $x = 5$. Substituting 5 in place of x in the original equation, we have

$$\begin{aligned} 5[2(5) - 1] &= 7(5) + 10 && \text{Substitute} \\ 5[10 - 1] &= 35 + 10 && \text{Order of operations} \\ 5[9] &= 45 \\ 45 &= 45 \quad \text{True} && \text{Solution checks} \end{aligned}$$

■ Example 2–1 C

Find the solution set.

1. $-2(y + 3) + 3(2y - 1) = 10$

$$\begin{aligned} -2y - 6 + 6y - 3 &= 10 && \text{Carry out the multiplication} \\ 4y - 9 &= 10 && \text{Combine like terms} \\ 4y - 9 + 9 &= 10 + 9 && \text{Add 9 to both members} \\ 4y &= 19 \\ \frac{4y}{4} &= \frac{19}{4} && \text{Divide both members by 4} \\ y &= \frac{19}{4} && \text{Solution} \end{aligned}$$

Check:

$$\begin{aligned} -2\left[\left(\frac{19}{4}\right) + 3\right] + 3\left[2\left(\frac{19}{4}\right) - 1\right] &= 10 && \text{Substitute} \\ -2\left[\frac{19}{4} + \frac{12}{4}\right] + 3\left[\frac{19}{2} - \frac{2}{2}\right] &= 10 && \text{Order of operations} \\ -2\left[\frac{31}{4}\right] + 3\left[\frac{17}{2}\right] &= 10 \\ -\frac{31}{2} + \frac{51}{2} &= 10 \\ \frac{20}{2} &= 10 \\ 10 &= 10 && \text{True} \quad \text{Solution checks} \end{aligned}$$

The solution set is $\left\{\frac{19}{4}\right\}$.

At this point, we will no longer show the check of our solution, but we should realize that a check of our work is an important final step.

The following equations contain several fractions. When this occurs, it is usually easier to **clear the equation of all fractions.** We do this by multiplying both members by the least common denominator (LCD) of all the fractions. Clearing all the fractions is considered a means of simplifying the equation and will be done as a first step when necessary. Equations containing fractions will be studied more completely in chapter 4.

2.
$$\frac{1}{2}x + 3 = \frac{2}{3}$$
$$6\left(\frac{1}{2}x + 3\right) = 6 \cdot \frac{2}{3}$$ Multiply both members by the least common denominator, 6
$$3x + 18 = 4$$
$$3x + 18 - 18 = 4 - 18$$ Subtract 18 from both members
$$3x = -14$$
$$\frac{3x}{3} = -\frac{14}{3}$$ Divide both members by 3
$$x = -\frac{14}{3}$$ Solution

The solution set is $\left\{-\frac{14}{3}\right\}$.

3.
$$\frac{3}{4}x - \frac{1}{2} = \frac{1}{3}x + 2$$
$$12\left(\frac{3}{4}x - \frac{1}{2}\right) = 12\left(\frac{1}{3}x + 2\right)$$ Multiply both members by the least common denominator, 12
$$9x - 6 = 4x + 24$$
$$9x - 4x - 6 = 4x - 4x + 24$$ Subtract 4x from both members
$$5x - 6 = 24$$
$$5x - 6 + 6 = 24 + 6$$ Add 6 to both members
$$5x = 30$$
$$\frac{5x}{5} = \frac{30}{5}$$ Divide both members by 5
$$x = 6$$ Solution

The solution set is $\{6\}$.

4.
$$3.18z + 3.526 = 2(0.73z - 2.709)$$
$$3.18z + 3.526 = 1.46z - 5.418$$ Carry out the multiplication
$$3.18z - 1.46z + 3.526 = 1.46z - 1.46z - 5.418$$ Subtract 1.46z from both members
$$1.72z + 3.526 = -5.418$$
$$1.72z + 3.526 - 3.526 = -5.418 - 3.526$$ Subtract 3.526 from both members
$$1.72z = -8.944$$
$$\frac{1.72z}{1.72} = \frac{-8.944}{1.72}$$ Divide both members by 1.72
$$z = -5.2$$ Solution

The solution set is $\{-5.2\}$.

5.
$$5(x - 4) - 2x = 3x + 7$$
$$5x - 20 - 2x = 3x + 7$$ Carry out the multiplication
$$3x - 20 = 3x + 7$$ Combine like terms
$$3x - 3x - 20 = 3x - 3x + 7$$ Subtract 3x from both members
$$-20 = 7$$ False

The statement $-20 = 7$ is false and this means that there is *no solution* to the equation. When an equation has no solution, it is called a **contradiction** and its solution set is $\emptyset$.

▶ ***Quick check*** Find the solution set. $4(5x - 2) + 7 = 5(3x + 1)$ ■

Problem solving

The following sets of word problems are designed to help us interpret verbal statements and write expressions for them in algebraic symbols. For each problem, we will write an algebraic expression that changes the words into mathematical symbols. These will not be equations. We will use our experience from these problems to help us translate word problems into equations in section 2–3.

Example 2–1 D

Write an algebraic phrase for each of the following verbal statements.

1. Phil can type 75 words per minute. How many words can he type in m minutes?

   If Phil can type 75 words in one minute, then we multiply

   $75 \cdot m = 75m$

   to obtain the number of words he can type in m minutes.

2. If Debbie has d dollars in her savings account and on successive days she deposits \$55 and then withdraws \$25 to make a purchase, write an expression for the balance in her savings account.

   We *add* the deposits and *subtract* the withdrawals. Thus

   $d + 55 - 25 = d + 30$

   represents the balance in dollars in Debbie's savings account after the two transactions.

3. A woman paid d dollars for 20 pounds of ground beef. How much did the beef cost her per pound?

   The price per pound is found by dividing the total cost by the number of pounds. Thus the price in dollars per pound of the beef is represented by

   $$\frac{d}{20}$$

Mastery points

Can you

- Apply the addition property of equality?
- Apply the multiplication property of equality?
- Solve linear equations?
- Determine when a linear equation has no solution?
- Check your solutions?
- Write an algebraic expression for a verbal statement?

Exercise 2–1

Find the solution set of the following linear equations. See examples 2–1 A, B, and C.

Example $4(5x - 2) + 7 = 5(3x + 1)$

Solution

$20x - 8 + 7 = 15x + 5$	Distributive property
$20x - 1 = 15x + 5$	Combine like terms
$20x - 15x - 1 = 15x - 15x + 5$	Subtract 15x
$5x - 1 = 5$	Combine like terms

$$
\begin{aligned}
5x - 1 + 1 &= 5 + 1 && \text{Add 1} \\
5x &= 6 && \text{Combine like terms} \\
\frac{5x}{5} &= \frac{6}{5} && \text{Divide by 5} \\
x &= \frac{6}{5} && \text{Solution}
\end{aligned}
$$

The solution set is $\left\{\frac{6}{5}\right\}$.

1. $5x = 15$
2. $7x = 28$
3. $4y = 10$
4. $6x = 27$
5. $a + 5 = 11$
6. $x + 4 = -3$
7. $z - 6 = -3$
8. $x - 9 = 8$
9. $\frac{x}{3} = 4$
10. $\frac{y}{2} = 11$
11. $\frac{3a}{4} = 8$
12. $4.1x = 15.17$
13. $1.8y = 21.6$
14. $0.7a = 11.2$
15. $\frac{2y}{3} = 9$
16. $3b - 1 = 8$
17. $5y - 2 = 13$
18. $4x + 5 = 5$
19. $6x + 4 = -2$
20. $9a - 3 = -3$
21. $7x - 4x + 3 = 8$
22. $2a + 3a - 7 = 5$
23. $3(y + 1) = 4$
24. $2(2z - 3) = 7$
25. $4(3b - 1) + 2b = 11$
26. $3(2x + 1) = 7x - 3x + 4$
27. $5(2 - x) + 1 = 3x - 4 + x$
28. $7x + 3 - 2x = 4(3 - 2x) + 5$
29. $2(3a - 1) = 3(2a + 5)$
30. $5(x - 4) + 12 = 3x + 2x - 6$
31. $\frac{1}{3}x + 2 = \frac{5}{6}$
32. $\frac{1}{2}x - 1 = \frac{1}{3}x + 3$
33. $\frac{3}{4}x + 3 = \frac{5}{8}x + 4$
34. $\frac{3}{8}x + \frac{1}{4} = \frac{1}{4}x + 3$
35. $\frac{5}{12}x + 2 = \frac{2}{3}x - 4$
36. $\frac{7}{12}x - 4 = \frac{5}{6}x - 5$
37. $-4(y - 3) + 2(3y - 5) = 11$
38. $-2(a + 3) + 5(a - 1) = -4$
39. $3(2y + 4) - 3y = 6(y + 1)$
40. $3(2x + 5) - x = 4(x - 3) + 7$
41. $3[2a - (a + 7)] = 10$
42. $4[a - (3a - 4)] = a + 5$
43. $-2[3x - (x - 5)] = 3x - 4$
44. $5.6z - 22.15 = 24.33$
45. $9.3y - 27.9 + 4.6y = 55.5$
46. $7.6a + 18.4 - 3.2a = 66.8$
47. $6.8x + 5.7 = 4.3x - 15.3$
48. $2.6(x - 6.3) = 8.9x - 81.9$
49. $6.7 - 4.1(x + 1) = 1.5x - 42.2$
50. $3(2x - 1) = 6x + 7$
51. $4a + 3a - 7 = 2(3a + 1) + a$
52. $6(z + 3) - 2z = 2(2z + 1)$

Solve the equations for the specified variable. See examples 2–1 A, B, and C.

53. The surface area S of a rectangular solid of length ℓ, width w, and altitude h is given by $S = 2(\ell w + \ell h + hw)$. Find ℓ when $S = 236$, $w = 5$, and $h = 6$.

54. To convert Celsius temperature to Fahrenheit, we use $F = \frac{9}{5}C + 32$. Find C when $F = 77$.

55. The amount A of a principal P invested for t years at simple interest with rate r percent per year is given by $A = P(1 + rt)$. Solve for t if $A = 4{,}080$; $P = 3{,}000$; and $r = 9\%$.

Write an algebraic expression for the following verbal statements. See example 2–1 D.

56. Rita can enter 80 words per minute on a word processor. How many words can she enter in m minutes?

57. Express the cost in cents of c cans of oil if each can costs \$1.15.

58. A 10-pound bag of dog food costs d dollars. How much does the dog food cost per pound?

59. It costs Pat \$18 to rent a posthole digger for h hours. What did it cost him per hour to rent the posthole digger?

60. Megan has n nickels, d dimes, and q quarters in her purse. Express in cents the amount of money she has in her purse. (*Hint:* n nickels are represented in cents by $5n$.)

61. Roger has h half dollars, q quarters, d dimes, and n nickels. Express in cents the amount of money Roger has.

62. Colleen is y years old now. Express her age (a) 21 years from now, (b) 7 years from now.

63. Richard is 3 years old. If Dick is n times as old as Richard, express Dick's age. Express Dick's age 8 years ago.

64. Edna's savings account has a current balance of \$457. If she makes a withdrawal of a dollars and then makes a deposit of b dollars, express her new balance.

65. Dale has a balance of d dollars in his checking account. He makes a deposit of \$464 and then writes 5 checks for m dollars each. Express his new balance in dollars in terms of d and m.

66. Marty has c cents, all in quarters. Write an expression for the number of quarters Marty has.

67. If w represents a whole number, write an expression for the next greater whole number.

68. If j represents an even integer, write an expression for the next greater even integer.

69. If j represents an odd integer, write an expression for the next greater odd integer.

70. If Joan is f feet and t inches tall, how tall is Joan in inches?

71. Sue earns \$500 more than twice the amount Jon earns in a year. If Jon earns d dollars in a year, write an expression for Sue's annual salary.

72. Lisa's annual salary is \$1,000 more than n times Bonnie's annual salary. If Bonnie earns \$9,000 per year, express Lisa's annual salary.

73. Express the total cost in cents of purchasing c cans of soda pop at 59¢ per can and b loaves of bread at \$1.15 per loaf.

74. A gallon of primer coat costs \$9.95 and a gallon of latex-base paint costs \$12.99. Express the cost of p gallons of primer and q gallons of latex-base paint.

75. Norm can type w words per minute and Rich can type 11 words per minute more than Norm. Write an expression for how many words Rich can type in 20 minutes.

Review exercises

Evaluate the following formulas. See section 1–5.

1. $F = ma$, $m = 24$ and $a = 11$

2. $I = prt$, $p = 5{,}000$; $r = 0.06$; and $t = 2$

3. $V_2 = V_1 + at$, $V_1 = 60$, $a = 32$, and $t = 5$

4. $A = \frac{1}{2}h(b_1 + b_2)$, $h = 6.2$, $b_1 = 4.5$, and $b_2 = 8$

5. $S = \frac{1}{2}gt^2$, $g = 32$ and $t = 4$

6. $\ell = a + (n - 1)d$, $a = 10$, $n = 15$, and $d = 2$

2–2 ■ Formulas and literal equations

In mathematics, a **literal equation** is an equation that contains more than one variable. A **formula** is a mathematical equation that states the relationship between two or more physical conditions.

When we repeatedly use a formula to determine the value of the same variable, it is convenient to solve the equation for that variable in terms of the remaining variables and constants. In the case of the relationship between Celsius and Fahrenheit temperature, it is useful to have one formula for Celsius in terms of Fahrenheit and another formula for Fahrenheit in terms of Celsius. Using our procedure for solving linear equations, we will now solve the Celsius formula for Fahrenheit F.

$$
\begin{aligned}
C &= \frac{5}{9}(F - 32) \\
9C &= 9 \cdot \frac{5}{9}(F - 32) && \text{Multiply both members by the least common denominator, 9} \\
9C &= 5(F - 32) \\
9C &= 5F - 160 && \text{Distributive property} \\
9C + 160 &= 5F && \text{Add 160 to both members} \\
\frac{1}{5}(9C + 160) &= \frac{1}{5} \cdot 5F && \text{Multiply both members by } \frac{1}{5} \\
\frac{9}{5}C + 32 &= F && \text{Distributive property}
\end{aligned}
$$

The formula for finding the temperature in degrees Fahrenheit, given the temperature in degrees Celsius, is

$$F = \frac{9}{5}C + 32$$

The two formulas

$$C = \frac{5}{9}(F - 32) \text{ and } F = \frac{9}{5}C + 32$$

may not look the same, but they express the same relationship between C and F. The first formula is solved for C in terms of F, and the second formula is solved for F in terms of C.

The following steps are a restatement of the procedure for solving linear equations. We will now apply these steps to solve formulas and literal equations for the specific variable.

Solving a literal equation or formula

Step 1 Simplify the equation.

Step 2 Obtain all terms with the variable for which we are solving in one member of the equation.

Step 3 Obtain all terms that do not have the variable for which we are solving in the other member of the equation.

Step 4 Determine the coefficient of the variable for which we are solving, and then divide both members of the equation by that coefficient.

Example 2–2 A

Solve for the specified variable.

1. Solve the literal equation $8x + 4 = 5x + y$, for x.

$$8x + 4 = 5x + y$$
$$8x - 5x + 4 = 5x - 5x + y \qquad \text{Subtract } 5x \text{ from both members}$$
$$3x + 4 = y$$
$$3x + 4 - 4 = y - 4 \qquad \text{Subtract 4 from both members}$$
$$3x = y - 4$$
$$\frac{3x}{3} = \frac{y - 4}{3} \qquad \text{Divide both members by 3}$$
$$x = \frac{y - 4}{3}$$

2. The formula for the perimeter of a rectangle is $P = 2\ell + 2w$. Solve for ℓ.

$$P = 2\ell + 2w$$
$$P - 2w = 2\ell + 2w - 2w \qquad \text{Subtract } 2w \text{ from both members}$$
$$P - 2w = 2\ell$$
$$\frac{P - 2w}{2} = \frac{2\ell}{2} \qquad \text{Divide both members by 2}$$
$$\ell = \frac{P - 2w}{2} \qquad \text{Symmetric property}$$

3. Solve the literal equation $2(3x - y) = 3(x + 3y) + 2$, for x.

$$2(3x - y) = 3(x + 3y) + 2$$
$$6x - 2y = 3x + 9y + 2 \qquad \text{Simplify each member}$$
$$6x - 3x - 2y = 3x - 3x + 9y + 2 \qquad \text{Subtract } 3x \text{ from both members}$$
$$3x - 2y = 9y + 2$$
$$3x - 2y + 2y = 9y + 2y + 2 \qquad \text{Add } 2y \text{ to both members}$$
$$3x = 11y + 2$$
$$\frac{3x}{3} = \frac{11y + 2}{3} \qquad \text{Divide both members by 3}$$
$$x = \frac{11y + 2}{3}$$

4. The area of a trapezoid is given by $A = \frac{1}{2}h\,(b_1 + b_2)$. Solve for b_1.

$A = \frac{1}{2}h(b_1 + b_2)$	
$2A = 2 \cdot \frac{1}{2}h(b_1 + b_2)$	Multiply both members by 2
$2A = h(b_1 + b_2)$	
$2A = b_1h + b_2h$	Distributive property
$2A - b_2h = b_1h$	Subtract b_2h from both members
$\frac{2A - b_2h}{h} = b_1$	Divide both members by h
$b_1 = \frac{2A - b_2h}{h}$	Symmetric property

Note Although we have not stated any restrictions on the variables, it is understood that the values that the variables can take on must be such that *no denominator is ever zero.* That is: examples 1, 2, and 3 have no restrictions; example 4, $h \neq 0$.

▶ ***Quick check*** Solve $P = 2\ell + 2w$, for w. ■

Whether we are solving a linear equation or a literal equation, the procedure is the same.

Linear equation	***Literal equation***	***Solve for* a**
$5(a + 1) = 2a + 7$	$5(a + b) = 2a + 7b$	Original equation
$5a + 5 = 2a + 7$	$5a + 5b = 2a + 7b$	Simplify (distributive property)
$3a + 5 = 7$	$3a + 5b = 7b$	All a's in one member
$3a = 2$	$3a = 2b$	Terms not containing a in other member
$a = \frac{2}{3}$	$a = \frac{2b}{3}$	Divide by the coefficient

In the linear equation, we have a solution for a, and in the literal equation, we have solved for a in terms of b.

Mastery points

Can you

- Solve formulas and literal equations for the specified variable in terms of the other variables?

Exercise 2–2

Find the value of the variable whose replacement value is not given.

Example $I = prt$. Solve for p if $I = 320$, $r = 0.08$, and $t = 2$.

Solution

$$\begin{aligned} I &= prt \\ 320 &= p(0.08)(2) && \text{Substitute} \\ 320 &= p(0.16) && \text{Simplify} \\ \frac{320}{0.16} &= \frac{p(0.16)}{0.16} && \text{Divide both members by (0.16)} \\ 2{,}000 &= p && \text{Simplify} \\ p &= 2{,}000 && \text{Symmetric property} \end{aligned}$$

1. $W = I^2R$; $I = 6$, $W = 324$

2. $L = \dfrac{V}{WH}$; $W = 12$, $H = 6$, $L = 20$

3. $A = P + Pr$; $A = 3{,}240$; $r = (0.08)$

4. $V = k + gt$; $k = 22$, $t = 3$, $V = 55$

5. $\ell = a + (n - 1)d$; $a = 7$, $d = 4$, $\ell = 83$

6. $A = \dfrac{1}{2}h(b_1 + b_2)$; $A = 54$, $b_2 = 10$, $h = 6$

7. $a = \dfrac{V_2 - V_1}{t}$; $a = 4$, $V_2 = 83$, $t = 8$

8. $T = \dfrac{R - R_0}{aR_0}$; $T = 2$, $R_0 = 4$, $a = 3$

Solve the following formulas or literal equations for the specified variable. Assume that no denominator is equal to zero. See example 2–2 A.

Example The formula for the perimeter of a rectangle is $P = 2\ell + 2w$. Solve for w.

Solution

$$\begin{aligned} P &= 2\ell + 2w \\ P - 2\ell &= 2\ell + 2w - 2\ell && \text{Subtract } 2\ell \text{ from both members} \\ P - 2\ell &= 2w && \text{Combine like terms} \\ \frac{P - 2\ell}{2} &= \frac{2w}{2} && \text{Divide both members by 2} \\ w &= \frac{P - 2\ell}{2} && \text{Symmetric property} \end{aligned}$$

9. $I = prt$; t

10. $E = IR$; R

11. $E = mc^2$; m

12. $V = \ell wh$; h

13. $F = ma$; m

14. $K = PV$; V

15. $A = bh$; b

16. $A = bh$; h

17. $W = I^2R$; R

18. $V = \ell wh$; w

19. $V = k + gt$; k

20. $P = 2\ell + 2w$; w

21. $V = k + gt$; g

22. $A = P + Pr$; r

23. $D = dq + R$; q

24. $m = -p(\ell - x)$; x

25. $m = -p(\ell - x)$; ℓ

$\boxed{26.}$ $R = W - b(2c + b)$; c

27. $R = W - b(2c + b)$; W

28. $2S = 2Vt - gt^2$; V

29. $V = r^2(a - b)$; a

30. $S = \dfrac{n}{2}[2a + (n - 1)d]$; a

31. $S = \dfrac{n}{2}[2a + (n - 1)d]$; d

32. $V = \dfrac{1}{3}\pi h^2(3R - h)$; R

33. $2S = 2Vt - gt^2$; g

34. $P = n(P_2 - P_1) - c$; P_1

35. $\ell = a + (n - l)d$; d

36. $2x + 3y = 12$; y

37. $2x + 3y = 12$; x

38. $2x - y = 5x + 6y$; x

39. $2x - y = 5x + 6y$; y

40. $3(4x - y) = 2x + y + 6$; y

41. $3(4x - y) = 2(x + y + 3)$; x

42. $ax + 4 = by - 3; x$
43. $ay + 7 - x = 3x + 4; y$
44. $a(x + 2) = by; x$
45. $b(y - 4) = a(x + 3); y$

Solve the following formulas or literal equations for the specified variable. Assume that no denominator is equal to zero. See example 2–2 A.

46. The distance s that a body projected downward with an initial velocity of v falls in t seconds because of the force of gravity is given by $s = \frac{1}{2}gt^2 + vt$. Solve for g.
47. Solve the formula in exercise 46 for v.
48. The net profit P on sales of n identical cars is given by $P = n(P_2 - P_1) - c$, where P_2 is the selling price, P_1 is the cost to the dealer, and c is the operating expense. Solve for P_2.
49. Solve the formula in exercise 48 for P_1.
50. The perimeter of an isosceles triangle with base b and sides s is given by $P = 2s + b$. Solve for s.

Review exercises

Write an algebraic expression for each of the following. See section 1–5.

1. The product of x and 3
2. 6 times the sum of a and 7
3. y decreased by 2 and that difference divided by 4
4. A number multiplied by 5
5. A number diminished by 12
6. A number divided by 8 and that quotient decreased by 9

2–3 ■ Word problems

Many problems that we encounter are written or stated verbally. We need to translate these word problems into equations that we can solve algebraically. When translating word problems into equations, we should look for phrases involving the basic operations of addition, subtraction, multiplication, and division. Table 1–1 in chapter 1 showed some examples of phrases that are commonly encountered.

We now combine our ability to write an expression and our ability to solve an equation and apply them for solving a word problem. While there is no standard procedure for solving a word problem, the following guidelines should be useful.

1. Read the problem carefully. Determine useful prior knowledge and note what information is given and what information we are asked to find.
2. Whenever possible, draw a picture or use a diagram to represent the information in the problem.
3. Let some letter represent one of the unknowns, then express other unknowns in terms of it.
4. Use the given conditions in the problem and the unknowns from step 3 to write an algebraic equation.
5. Solve the equation for the unknown. Relate this answer to any other unknowns in the problem.
6. Check the results in the original statement of the problem.

Example 2–3 A

Number problems

Write an equation for the problem and solve for the unknown quantities.

1. One number is 18 more than a second number. If their sum is 62, find the two numbers.

   If we knew the value of the second number, then the first number would be 18 more. Therefore let x be the second number (second number $= x$). Then the first number is $x + 18$. Since these two numbers add up to 62, we write the equation as

first number	sum	second number	is	62
$(x + 18)$	$+$	x	$=$	62

$$
\begin{aligned}
(x + 18) + x &= 62 && \text{Equation}\\
x + 18 + x &= 62 && \text{Remove parentheses}\\
2x + 18 &= 62 && \text{Combine like terms}\\
2x &= 44 && \text{Subtract 18 from both members}\\
x &= 22 && \text{Divide both members by 2}\\
\text{and } x + 18 = (22) + 18 &= 40 && \text{Substitute to determine the other number}
\end{aligned}
$$

   Hence the second number is 22 and the first number is 40.

   To check our answers, we must determine whether they satisfy the conditions stated in the original problem. Since 40 is 18 more than 22 and since the sum of 22 and 40 is 62, we know that our answers are correct. We will not show the checks of the following problems, but we should realize that a check of our work is an important final step.

▶ ***Quick check*** One number is 9 times a second number and their sum is 120. Find the numbers.

2. If a number is divided by 4 and this quotient is increased by 6, the result is 13. Find the number.

   Let x be the number. Then the number divided by 4 is $\frac{x}{4}$ and that quotient increased by 6 is $\frac{x}{4} + 6$. Since this equals 13, we have

number divided by 4	increased	by 6	is	13
$\frac{x}{4}$	$+$	6	$=$	13

   Solving for x,

$$
\begin{aligned}
\frac{x}{4} + 6 &= 13 && \text{Equation}\\
\frac{x}{4} &= 7 && \text{Subtract 6 from both members}\\
x &= 28 && \text{Multiply both members by 4}
\end{aligned}
$$

   Hence the number is 28.

3. The sum of three numbers is 63. The first number is twice the second number, and the third number is three times the first number. Find the three numbers.

We must know the value of the second number to find the first number, and we must know the value of the first number to find the third number. Therefore everything depends on the second number. If x equals the second number, we have

second number $= x$
first number $= 2x$ (twice the second)
third number $= 3(2x) = 6x$ (three times the first).

Since their sum is 63, we have

$$\underset{\text{first number}}{2x} \underset{\text{sum}}{+} \underset{\text{second number}}{x} \underset{\text{sum}}{+} \underset{\text{third number}}{6x} \underset{\text{is}}{=} \underset{63}{63}$$

Solving for x,

$$\begin{aligned} 2x + x + 6x &= 63 && \text{Equation} \\ 9x &= 63 && \text{Combine like terms} \\ x &= 7 && \text{Divide by 9} \end{aligned}$$

Hence, $2x = 2(7) = 14$ and $6x = 6(7) = 42$. Therefore the second number (x) is 7, the first number ($2x$) is 14, and the third number ($6x$) is 42.

▶ ***Quick check*** The sum of three consecutive odd integers is 51. Find the integers.

Interest problem

4. Phil had \$20,000, part of which he invested at 8% interest and the rest at 6%. If his total income from the two investments for one year was \$1,460, how much did he invest at each rate?

To solve this problem, we must understand how interest is computed. If we invested \$5,000 at 8% interest, after one year we would have 8% of \$5,000 or $(0.08)(\$5{,}000) = \$400$ interest. Thus the amount of interest earned in one year is the product of the rate times the principal (the amount invested).

In the given problem, we have two principals and two rates, and our formula will involve the sum of the two earned interests, which is equal to \$1,460.

Let x represent the amount of money invested at 8%. Since this amount and the amount invested at 6% total \$20,000, we can describe the 6% principal as the remainder after the amount x has been invested at 8%, that is, $20{,}000 - x$ is invested at 6% interest. We can use a table to summarize the information.

	Investment earning 8%	**Investment earning 6%**	**Total**
Amount invested	x	$20{,}000 - x$	20,000
Interest received	$(0.08)x$	$(0.06)(20{,}000 - x)$	1,460

Mastery points

Can you

- Translate word problems into equations?
- Solve for the unknown quantities?

Exercise 2–3

Write an equation for the problem and solve for the unknown quantities. See example 2–3 A–1, 2, and 3.

Number problems

Example One number is 9 times a second number and their sum is 120. Find the numbers.

Solution Let x represent the second number, then the first number is 9 times the second number, or $9x$. Since their sum is 120, the equation is

first number	sum	second number	is	120
$9x$	$+$	x	$=$	120

Solving for x,

$9x + x = 120$ Equation

$10x = 120$ Combine like terms

$x = 12$ Divide by 10

and $9x = 9(12) = 108$ Substitute to get other number

Therefore the second number (x) is 12 and the first number ($9x$) is 108.

1. One number is 8 more than a second number. If their sum is 88, find the two numbers.
2. One whole number is 6 times a second whole number and their sum is 63. Find the numbers.
3. One natural number is 8 times another natural number and their sum is 54. Find the natural numbers.
4. One number is 28 more than a second number. If their sum is 62, find the two numbers.
5. One number is 5 less than another number. If their sum is 47, find the two numbers.
6. The difference of two numbers is 17. Find the numbers if their sum is 85.
7. If three times a number is increased by 11 and the result is 65, what is the number?
8. Nine times a number is decreased by 4, leaving 122. What is the number?
9. If a number is divided by 4 and that result is then increased by 6, the answer is 27. Find the number.
10. One-half of a number minus one-third of the number is 8. Find the number.
11. If a number is decreased by 14 and that result is then divided by 5, the answer is -8. Find the number.
12. One-third of a number is 12 less than one-half of the number. Find the number.
13. What number added to its double gives 51?
14. Find a number such that twice the sum of that number and 7 is 38.
15. Find two numbers whose sum is 81 and whose difference is 35.
16. One number is 11 more than twice a second number. If their sum is 53, what are the numbers?
17. Six times a number, increased by 10, gives 88. Find the number.
18. One number is seven times another. If their difference is 28, what are the numbers?
19. The sum of the number of teeth on two gears is 64 and their difference is 12. How many teeth are on each gear?
20. Two gears have a total of 83 teeth. One gear has 15 less teeth than the other. How many teeth are on each gear?

21. Two electrical voltages have a total of 126 volts (V). If one voltage is 32 V more than the other, find the voltages.

22. The sum of two voltages is 85 and their difference is 32. Find the voltages.

23. The sum of two resistances in a series is 24 ohms and their difference is 14 ohms. How many ohms are in each resistor?

24. One resistor exceeds another resistor by 25 ohms and their sum is 67 ohms. How many ohms are in each resistor?

Example The sum of three consecutive odd integers is 51. Find the integers.

Solution First we shall examine consecutive odd integers in general. Consider the list 1, 3, 5, 7 or 215, 217, 219, 221. We observe that the next odd integer on either list is found by adding 2 to the previous integer. Therefore if we let x be the first (least) of the three consecutive odd integers, then $x + 2$ would be the second and $(x + 2) + 2 = x + 4$ must be the third. Since their sum is 51, the equation would be

first odd integer	sum	second odd integer	sum	third odd integer	is	51
x	$+$	$(x + 2)$	$+$	$x + 4$	$=$	51

Solving for x,

$$x + x + 2 + x + 4 = 51 \quad \text{Remove grouping symbols}$$
$$3x + 6 = 51 \quad \text{Combine like terms}$$
$$3x = 45 \quad \text{Subtract 6}$$
$$x = 15 \quad \text{Divide by 3}$$

Therefore $x + 2 = (15) + 2 = 17$ and $x + 4 = (15) + 4 = 19$

Hence the first consecutive odd integer (x) is 15, the second $(x + 2)$ is 17, and the third $(x + 4)$ is 19.

25. The sum of three consecutive integers is 69. Find the integers.

26. The sum of three consecutive integers is 93. Find the integers.

27. The sum of three consecutive even integers is 48. Find the integers.

28. The sum of three consecutive even integers is -48. Find the integers.

29. The sum of three consecutive odd integers is -63. Find the integers.

30. The sum of three consecutive odd integers is 87. Find the integers.

31. One number is 27 more than another. The smaller number is one-fourth of the larger number. Find the numbers.

32. A number plus one-half of the number plus one-third of the number equals 33. Find the number.

33. A number is decreased by 7 and twice this result is 34. What is the number?

34. The sum of three numbers is 100. The second number is three times the first number and the third number is 6 less than the first number. Find the three numbers.

35. One number is 11 more than another number. Find the two numbers if three times the larger number exceeds four times the smaller number by 4.

36. One number is 8 more than another number. Find the two numbers if two times the larger number is 11 less than five times the smaller number.

37. If the first of two consecutive integers is multiplied by 3, this product is 20 more than the sum of the two integers. Find the integers.

38. Four times the first of three consecutive integers is 1 less than three times the sum of the second and third. Find the integers.

39. Five times the first of three consecutive even integers is 4 less than twice the sum of the second and third. Find the integers.

40. One-fourth of the middle integer of three consecutive even integers is 24 less than one-half of the sum of the other two integers. Find the three integers.

41. The sum of three numbers is 49. The second number is three times the first number and the third number is 6 less than the first number. Find the three numbers.

42. The sum of three numbers is 38; the second number is twice the first number and the third number is 2 more than the first number. Find the three numbers.

43. One number is 7 more than another number. Find the two numbers if three times the larger number exceeds four times the smaller number by 13.

Interest problems

See example 2–3 A–4.

Example Lynne made two investments totaling \$25,000. She made an 18% profit on one investment, but she took an 11% loss on the other investment. If her net gain was \$2,180, how much was invested at each rate?

Solution Let x be the amount invested at 18% profit, then the amount invested at 11% loss was what was left over from the \$25,000 or $25{,}000 - x$. Her profit of 18% on the one investment is denoted by $(0.18)x$ and her loss of 11% on the other investment is $-(0.11)(25{,}000 - x)$; the loss is denoted as a negative amount. We can use a table to summarize the information in the problem.

	Investment earning 18%	Investment losing 11%	Total
Amount invested	x	$25{,}000 - x$	25,000
Profit or loss	$(0.18)x$	$-(0.11)(25{,}000 - x)$	2,180

We get the equation for the problem from the bottom row of the table.

$$\underbrace{(0.18)x}_{\text{18\% profit}} \underbrace{-}_{\text{net}} \underbrace{(0.11)(25{,}000 - x)}_{\text{11\% loss}} \underbrace{=}_{\text{was}} \underbrace{2{,}180}_{2{,}180}$$

Solving for x,

$$\begin{aligned}
(0.18)x - (0.11)(25{,}000 - x) &= 2{,}180 && \text{Equation}\\
(0.18)x - 2{,}750 + (0.11)x &= 2{,}180 && \text{Distributive property}\\
(0.29)x - 2{,}750 &= 2{,}180 && \text{Combine like terms}\\
(0.29)x &= 4{,}930 && \text{Add 2,750}\\
x &= 17{,}000 && \text{Divide by 0.29}\\
\text{and } 25{,}000 - x = 25{,}000 - (17{,}000) &= 8{,}000 && \text{Substitute to get other investment}
\end{aligned}$$

So the amount invested at an 18% profit (x) was \$17,000 and the amount invested at an 11% loss ($25{,}000 - x$) was \$8,000.

44. Robert had \$37,000, part of which he invested at 8% interest and the rest at 6%. If his total income was \$2,600 from the two investments, how much did he invest at each rate?

45. Terry has \$15,000. He invests part of this money at 8% and the rest at 6%. His income for one year from these investments totals \$1,100. How much is invested at each rate?

46. Tammy had \$6,000. She invested part of her money at $7\frac{1}{2}\%$ interest and the rest at 9%. If her income from the two investments was \$511.50, how much did she invest at each rate?

47. Harry invested \$26,000, part at 10% and the rest at 12%. If his income for one year from these investments totals \$2,860, how much was invested at each rate?

48. Jill invests a total of $12,000, part at 6% and part at 10%. If her total income for one year is $956, how much is invested at each rate?

49. Margot invests a total of $12,000, part at 10% and part at 12%. Her total income for one year from the investments totals $1,340. How much is invested at each rate?

50. Alanzo has $16,875, part of which he invests at 10% interest and the rest at 8%. If his income from each investment is the same, how much did he invest at each rate?

51. Andrew invested a total of $18,000, part at 5% and part at 9%. If his income for one year from the 9% investment was $100 less than his income from the 5% investment, how much was invested at each rate?

52. Peter has invested $5,000 at an 8% rate. How much more must he invest at 10% to make the total income for one year from both sources a 9% rate?

53. Sherri has $4,000 invested at 7% and is going to invest an additional amount at 11% so that her total investment will make 9%. How much does she need to invest at 11% to achieve this?

54. Jeremy has $6,000 invested at 6%; how much must he invest at 10% to realize a net return of 9%?

55. Dick has $26,000, part of which he invests at 10% interest and the rest at 14%. If his income for one year from the 14% investment is $760 more than that from the 10% investment, how much is invested at each rate?

56. Susan had $19,000, part of which she invested at 9% interest and the rest at 7%. If her income from the 7% investment was $40 more than that from the 9% investment, how much did she invest at each rate?

57. Nina made two investments totaling $18,000. She made a 14% profit on one investment, but she took a 9% loss on the other investment. If her net gain was $680, how much was each investment?

58. Donald made two investments totaling $17,500. One investment made him a 13% profit, but on the other investment he took a 9% loss. If his net loss was $475, how much was each investment?

59. Jim made two investments totaling $34,000. One investment made him a 12% profit, but on the other investment he took a 21% loss. If his net loss was $870, how much was each investment?

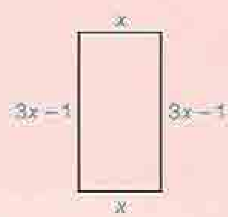
Geometry problems

See example 2–3 A–5.

Example The length of a rectangle is 1 inch less than three times the width. Find the dimensions if the perimeter is 70 inches.

Solution If x represents the width of the rectangle, then by multiplying the width by three $(3x)$ and subtracting 1, $(3x - 1)$, represents the length of the rectangle. Using the formula for the perimeter of a rectangle, $P = 2\ell + 2w$, and the fact that the perimeter P is 70, we can write the equation

$P = 2\ell + 2w$ Formula for perimeter
$70 = 2(3x - 1) + 2x$ Substitute



Solving for x,

$70 = 6x - 2 + 2x$ Equation
$70 = 8x - 2$ Combine like terms
$72 = 8x$ Add 2
$9 = x$ Divide by 8
and $3x - 1 = 3(9) - 1 = 27 - 1 = 26$ Substitute

Therefore the width of the rectangle (x) is 9 inches and the length $(3x - 1)$ is 26 inches.

60. The length of a rectangle is 9 feet more than its width. The perimeter of the rectangle is 90 feet. Find the dimensions.

61. The length of a rectangle is 5 feet more than its width. If the perimeter is 102 feet, find the length and width.

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62. The width of a rectangle is 3 feet less than its length. The perimeter of the rectangle is 110 feet. Find the dimensions.

63. The width of a rectangle is $\frac{1}{3}$ of its length. If the perimeter is 128 feet, find the dimensions.

64. The width of a rectangle is 3 meters less than the length. If the perimeter of the rectangle is 126 meters, find the dimensions of the rectangle.

65. One side of a triangle is twice as long as the second side and the third side is 4 less than three times the second side. If the perimeter is 38 centimeters, find the lengths of the sides.

66. One side of a triangle is three times as long as the second side and the third side is 1 more than two times the second side. If the perimeter is 37 meters, find the lengths of the sides.

Mixture problems

Example What quantities of 65% pure silver and 45% pure silver must be mixed together to give 100 grams of 50% pure silver?

Solution To solve this mixture problem, we need to understand that the amount of silver in any given mixture is found by multiplying the percent of silver in the mixture times the amount of mixture. If x represents the amount of 65% pure silver in the final mixture, then $100 - x$ will represent the amount of 45% pure silver. The following table summarizes the information in the problem.

	65% silver mixture	45% silver mixture	50% silver mixture
Number of grams	x	$100 - x$	100
Amount of silver	$(0.65)x$	$(0.45)(100 - x)$	$(0.50)(100)$

We get the equation for the problem from the bottom row of the table.

$$\underbrace{(0.65)x}_{\text{amount of silver in 65\% pure}} \underbrace{+}_{\text{mixed together}} \underbrace{(0.45)(100 - x)}_{\text{amount of silver in 45\% pure}} \underbrace{=}_{\text{to give}} \underbrace{(0.50)(100)}_{\text{100 grams of 50\% pure silver}}$$

Solving for x,

$$\begin{aligned}
(0.65)x + (0.45)(100 - x) &= (0.50)(100) && \text{Equation}\\
(0.65)x + 45 - (0.45)x &= 50 && \text{Multiply}\\
(0.20)x + 45 &= 50 && \text{Combine like terms}\\
(0.20)x &= 5 && \text{Subtract 45}\\
x &= 25 && \text{Divide by 0.20}\\
\text{and } 100 - x = 100 - (25) &= 75 && \text{Substitute to get other amount}
\end{aligned}$$

So the amount of 65% pure silver (x) is 25 grams and the amount of 45% pure silver ($100 - x$) is 75 grams.

67. An auto mechanic has two bottles of battery acid solutions. One contains 10% acid and the other 4% acid. How many cubic centimeters of each solution must be used to make 120 cm^3 of a solution that is 6% acid?

68. A metallurgist wishes to form 2,000 kg of an alloy that is 80% copper. This alloy is to be obtained by fusing some alloy that is 68% copper and some alloy that is 83% copper. How many kilograms of each alloy must be used?

69. If a jeweler wishes to form 12 ounces of 75% pure gold from substances that are 60% and 80% pure gold, how much of each substance must be mixed together to produce this?

70. A chemist wishes to make 1,000 liters of a 3.5% acid solution by mixing a 2.5% solution with a 25% solution. How many liters of each solution is necessary?

71. A pharmacist wishes to fill a total of 200 3-grain and 2-grain capsules using 500 grains of a certain drug. How many capsules of each kind does he fill?

72. A solution that is 38% silver nitrate is to be mixed with a solution that is 3% silver nitrate to obtain 100 centiliters of solution that is 5% silver nitrate. How many centiliters of each solution should be used in the mixture?

73. A druggist has two solutions, one 60% hydrogen peroxide and the other 30% hydrogen peroxide. How many liters of each should she mix to obtain 30 liters of a solution that is 40% hydrogen peroxide?

Review exercises

Write the values of the following numbers. See section 1–1.

1. $|-12|$ 2. $|0|$ 3. $\left|-\frac{3}{4}\right|$ 4. $|6|$

Find the solution set. See section 2–1.

5. $3x - 6 = 14$ 6. $2x + 5 = -7$ 7. $4 - 3x = -11$ 8. $3x - 2 = 4x + 5$

2–4 ■ Equations involving absolute value

Absolute value equations

In chapter 1, we defined the absolute value of a number x to be

$$|x| = \begin{cases} x, \text{ if } x \geq 0 \\ -x, \text{ if } x < 0 \end{cases}$$

The absolute value of a number represents the undirected distance from that number to the origin on the number line, that is, the distance from x to 0.

Consider the equation

$$|x| = 2$$

The right member, 2, denotes the distance that the graph of x is located from the origin. The equation directs us to find all numbers that are 2 units from the origin.

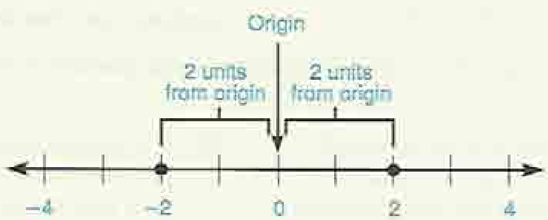


Figure 2–1

We see from figure 2–1 that 2 and -2 are both 2 units from the origin and, therefore, satisfy the equation $|x| = 2$. The solution set is then given as $\{-2,2\}$.

$|x| = a$

For any real number x and $a \geq 0$,

$$|x| = a \text{ is equivalent to } x = a \text{ or } x = -a$$

Concept

An equation of the form $|x| = a$, called an **absolute value equation,** is equivalent to the equations $x = a$ or $x = -a$.

Note Recall that $|x| \geq 0$. We realize from this that a in our generalizations must be nonnegative, $a \geq 0$. This means that in an equation of this type, there is no solution if a is negative. For example, if $|x| = -2$, then the solution set is $\emptyset$.

Solving absolute value equations

1. Isolate the absolute value in one member of the equation.
2. Write the two equivalent equations.
3. Solve each equation.
4. Check each answer in the original absolute value equation.

Solutions for the absolute value equation $|x| = a$

If a is positive, the solutions are $x = a$ or $x = -a$	If a is zero, the solution is $x = 0$	If a is negative, there is no solution.

■ *Example 2–4 A*

Find the solution set of the following absolute value equations.

1. $|x| = 5$

$x = 5$ or $x = -5$ — Write the two equivalent equations

The solution set is $\{-5,5\}$.

To check the solutions of an absolute value equation, simply substitute the solutions into the original equation. In example 1, this would be:

For 5	For −5
$\lvert 5\rvert = 5$	$\lvert -5\rvert = 5$
$5 = 5$ True	$5 = 5$ True

2. $|x| + 4 = 10$

$|x| = 6$ — **Isolate** the absolute value by subtracting 4

$x = 6$ or $x = -6$ — Write the two equivalent equations

The solution set is $\{-6,6\}$.

3. $|x + 5| = 8$

$x + 5 = 8$ or $x + 5 = -8$ — Write the two equivalent equations

$x = 3$ $\qquad$ $x = -13$ — Subtract 5 from both members

The solution set is $\{-13,3\}$.

4. $|3a - 2| = 7$

$$\begin{array}{rclcrcll} 3a - 2 &=& 7 & \text{or} & 3a - 2 &=& -7 & \text{Write the two equivalent equations} \\ 3a &=& 9 & & 3a &=& -5 & \text{Add 2 to both members of both equations} \\ a &=& 3 & & a &=& -\frac{5}{3} & \text{Divide both members of both equations by 3} \end{array}$$

The solution set is $\left\{-\frac{5}{3}, 3\right\}$.

5. $|4x - 3| + 7 = 5$

$$|4x - 3| = -2 \qquad \text{Isolate the absolute value in one member}$$

The solution set is $\emptyset$. Since the absolute value of any quantity cannot be negative, the solution set is empty

6. $|2a - 3| = |a + 4|$

The solution set to this equation will be those values that satisfy the condition that either $2a - 3$ and $a + 4$ are equal to each other or are opposites of each other. Hence we have the following equations.

$$\overbrace{2a - 3 = a + 4}^{\text{Equal to each other}} \quad \text{or} \quad \overbrace{2a - 3 = -(a + 4)}^{\text{Opposites of each other}}$$

Solving each of the equations, we have

$$\begin{array}{rcll rcll} 2a - 3 &=& a + 4 & & 2a - 3 &=& -(a + 4) & \\ a - 3 &=& 4 & \text{Subtract } a & 2a - 3 &=& -a - 4 & \text{Remove parentheses} \\ a &=& 7 & \text{Add 3} & 3a - 3 &=& -4 & \text{Add } a \\ & & & & 3a &=& -1 & \text{Add 3} \\ & & & & a &=& -\frac{1}{3} & \text{Divide by 3} \end{array}$$

$$\left\{-\frac{1}{3}, 7\right\} \qquad \text{Solution set}$$

7. $|2x + 3| = |2x - 4|$

We form the two equations equivalent to the absolute value equation.

$$\overbrace{2x + 3 = 2x - 4}^{\text{Equal to each other}} \quad \text{or} \quad \overbrace{2x + 3 = -(2x - 4)}^{\text{Opposites of each other}}$$

Solving each of the equations, we have

$$\begin{array}{rcll rcll} 2x + 3 &=& 2x - 4 & & 2x + 3 &=& -(2x - 4) & \\ & & & \text{Subtract } 2x & 2x + 3 &=& -2x + 4 & \text{Remove parentheses} \\ 3 &=& -4 & \text{False} & 4x + 3 &=& 4 & \text{Add } 2x \\ & & & & 4x &=& 1 & \text{Subtract 3} \\ & & & & x &=& \frac{1}{4} & \text{Divide by 4} \end{array}$$

Since the first equation is a false statement ($3 = -4$), the first equation has no solution. The solution set is only the answer to the second equation and is given by $\left\{\frac{1}{4}\right\}$.

Note In examples 6 and 7, the solution set of the absolute value equation is the same whether we take the opposite of the first expression or the opposite of the second expression. That is, our solution set would be the same if we had solved $-(2a - 3) = a + 4$ instead of $2a - 3 = -(a + 4)$, or $-(2x + 3) = 2x - 4$ instead of $2x + 3 = -(2x - 4)$.

▶ ***Quick check*** Find the solution set. $|4b + 3| = 5$ ■

Mastery points

Can you

- Solve absolute value equations?

Exercise 2–4

Determine whether or not the given value is a solution of the absolute value equation. Use $|2x + 7| = 11$ for exercises 1–4 and $|3x - 1| = 10$ for exercises 5–8.

Example Is -9 a solution of the absolute value equation $|2x + 7| = 11$?

Solution

$$\begin{aligned} |2(-9) + 7| &= 11 && \text{Substitute} \\ |-18 + 7| &= 11 \\ |-11| &= 11 && \text{True} \end{aligned}$$

Therefore -9 is a solution of $|2x + 7| = 11$.

1. 2 **2.** -2 **3.** $\frac{1}{2}$ **4.** $\frac{5}{4}$ **5.** 3 **6.** -3 **7.** $\frac{11}{3}$ **8.** $\frac{2}{3}$

Find the solution set of the following absolute value equations. See example 2–4 A.

Example $|4b + 3| = 5$

Solution

$$\begin{aligned} 4b + 3 &= 5 \quad \text{or} \quad & 4b + 3 &= -5 && \text{Write the equivalent equations} \\ 4b &= 2 & 4b &= -8 && \text{Subtract 3 from all members} \\ b &= \frac{1}{2} & b &= -2 && \text{Divide all members by 4} \end{aligned}$$

The solution set is $\left\{-2, \frac{1}{2}\right\}$.

9. $|x| = 9$ **10.** $|x| = 12$ **11.** $|a| = 4$ **12.** $|b| = 5$

13. $|b| + 2 = 6$ **14.** $|x| + 4 = 10$ **15.** $|x| - 5 = 7$ **16.** $|y| - 8 = 2$

17. $|x + 4| = 6$ **18.** $|x + 3| = 6$ **19.** $|a - 3| = 2$ **20.** $|b - 7| = 11$

21. $|3x - 4| = 8$ **22.** $|2x - 1| = 9$ **23.** $|2a + 7| = 9$ **24.** $|3x + 2| = 11$

25. $|5x - 3| = -4$ **26.** $|x + 2| = -1$ **27.** $|4a + 8| + 10 = 3$ **28.** $|3b + 1| + 8 = 5$

29. $|4b - 3| + 2 = 8$ **30.** $|2x + 5| + 3 = 10$ **31.** $|5a + 2| - 7 = 4$ **32.** $|3y - 2| + 4 = 11$

33. $|2.1x - 6.3| = 8.4$
34. $|3.2x - 6.4| = 9.6$
35. $|1.8x - 10.8| = 5.4$
36. $|1.7a - 5.1| = 13.6$
37. $\left|\frac{1}{2}x + 5\right| = 7$
38. $\left|\frac{1}{3}x - 2\right| = 13$
39. $\left|\frac{3}{4}a - 2\right| = 6$
40. $\left|\frac{2}{3}x + 1\right| = 10$
41. $\left|\frac{2}{3}b - 6\right| = 4$
42. $\left|\frac{2}{5}x - 3\right| = 17$
$\boxed{43.}$ $|5 - 3x| - 4 = 8$
44. $|2 - x| = 8$
45. $|3 - 4a| + 2 = 5$
46. $|1 - 2x| = 21$
$\boxed{47.}$ $|3x - 7| = |5x + 3|$
48. $|2x + 1| = |x - 3|$
49. $|2a + 5| = |6a + 7|$
50. $|x + 5| = |3x - 4|$
51. $|3 - 2a| = |4a + 6|$
52. $|1 - 3b| = |2b + 3|$
53. $|3y - 4| = |3y + 8|$
54. $|3x - 2| = |3x + 4|$
55. $|4b - 3| = |4b + 7|$
56. $|2y + 5| = |2y - 7|$

Write an absolute value equation for the following statements and solve for the unknown.

Example The absolute value of twice a number is 10.

Solution If we let x represent the unknown number, then twice the number would be $2x$. The absolute value equation would be

$|2x| = 10$ — Equation

$2x = 10$ or $2x = -10$ — Write the two equivalent equations

$x = 5$ $\quad x = -5$ — Divide both members of both equations by 2

The solution set is $\{-5,5\}$.

57. The absolute value of a number is 6.
58. The absolute value of a number is 9.
59. The absolute value of 3 times a number is 12.
60. The absolute value of 5 times a number is 30.
61. If a number is diminished by 8, the absolute value of the result is 14.
62. If a number is diminished by 6, the absolute value of the result is 12.
63. If a number is increased by 7, the absolute value of the result is 15.
64. Twice a number is increased by 3. The absolute value of the result is 11.
65. Three times a number is diminished by 4. The absolute value of the result is 8.
66. One-half of a number is added to 3. The absolute value of the result is 4.
67. One-third of a number is diminished by 7. The absolute value of the result is 14.
68. One-half of a number is diminished by 5. The absolute value of the result is 11.

Review exercises

Write an algebraic expression for each of the following. See section 1–5.

1. The sum of x and 2
2. 6 less than y
3. a diminished by 4
4. A number divided by 5
5. $\frac{1}{3}$ of a number
6. A number decreased by 2 and that difference divided by 8

2–5 ■ Linear inequalities

Representing the solution set of a linear inequality

When we replace the equals sign in a linear equation with one of the inequality symbols ($<$ is less than, $>$ is greater than, $\leq$ is less than or equal to, $\geq$ is greater than or equal to), we form a **linear inequality.** The following are examples of linear inequalities.

$$5x + 3 \leq 4, \quad 2(3x - 1) > 5x - 7, \quad 5x - 2x + 1 < x - 3$$

A major difference between a linear equation and a linear inequality is the solution set. The solution set of a conditional linear equation has at most one solution, whereas the solution set of a conditional linear inequality usually consists of an unlimited number of solutions. Consider the inequality $3x \geq 9$. We see by inspection that if we substitute 3, $3\frac{1}{3}$, 4, or 5 for x, the inequality would be true. In fact, we see that if we substitute any number greater than or equal to 3, that is ($x \geq 3$), the inequality would be true. This demonstrates that our solution set has an unlimited number of solutions. We would state it in set-builder notation as $\{x|x \geq 3\}$, which is read "the set of all elements x such that x is greater than or equal to 3."

There are other ways to indicate the solution set of an inequality. One way is to graph the solution set. To graph the solution set $\{x|x \geq 3\}$, we simply draw a number line (as we did in chapter 1), place a *solid circle* (dot) at 3 on the number line, and draw an arrow extending from the circle to the right, as in figure 2–2.

Figure 2–2

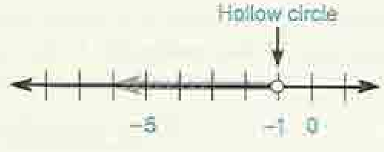
The solid circle at 3 represents the fact that 3 is included in the solution set.

■ Example 2–5 A

Represent the following solution sets graphically.

1. $\{x|x < -1\}$

   Here x is representing all real numbers less than -1, but not -1 itself. To denote the fact that x cannot equal -1, we put a *hollow circle* at -1.

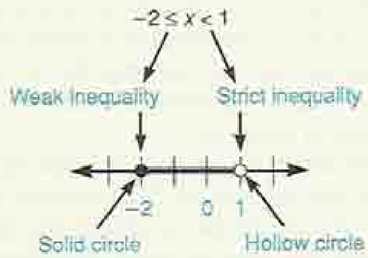


2. $\{x | x \geq 2\}$

The greater than or equal to symbol, $\geq$, indicates that 2 is included in the solution set, so we place a solid circle at 2.

3. $\{x | -2 \leq x < 1\}$

The statement $-2 \leq x < 1$ is called a **compound inequality** and is read "-2 is less than or equal to x and x is less than 1." We place a solid circle at -2 to denote that -2 is included, and we place a hollow circle at 1 to show that 1 is not included. We then draw a line segment between the two circles.



Note When we graph inequalities, a strict inequality, $<$ or $>$, is represented by a hollow circle at the number. A weak inequality, $\leq$ or $\geq$, is represented by a solid circle at the number.

▶ ***Quick check*** Represent the solution set graphically. $\{x | -3 < x \leq 2\}$ ■

Interval notation

Another way to represent the solution set of an inequality is to use **interval notation.** In interval notation we use a parenthesis, (or), to represent that the endpoint is *not* part of the interval and a bracket, [or], to represent that the endpoint is part of the interval.

Consider the set $\{x | -3 < x \leq 4\}$. To write this in interval notation, we first write down the endpoints, writing the lesser one first, and separate them by a comma.

$$-3,4$$

If the endpoint is indicated by a strict inequality symbol, $<$ or $>$, place a parenthesis next to that endpoint. If the endpoint is indicated by a weak inequality symbol, $\leq$ or $\geq$, place a bracket next to that endpoint. So,

Strict inequality—parenthesis

$\{x | -3 < x \leq 4\}$ is equivalent to $(-3,4]$

Weak inequality—bracket

Example 2–5 B

Represent the following sets with interval notation.

1. $\{x|-2 \le x \le 3\}$

   Since the endpoints are included, we use brackets in interval notation.

   $[-2,3]$

2. $\{x|0 < x < 6\}$

   Since the endpoints are not included, we use parentheses in interval notation.

   $(0,6)$

3. $\{x|x \ge 1\}$

   To indicate that our solution set continues indefinitely, we use $+\infty$, read "positive infinity." We place a parenthesis next to the $+\infty$ symbol because there is no endpoint to be contained.

   $[1,+\infty)$

4. $\{x|x < -3\}$

   Interval $(-\infty,-3)$
   The symbol $-\infty$ is read "negative infinity."

Note In interval notation, the lesser number must come first. In example 4, $(-3,-\infty)$ would be incorrect.

▶ ***Quick check*** Represent the set $\{x|-3 < x \le 2\}$ with interval notation.

Solving a linear inequality

The properties that we will use to solve a linear inequality are similar to those that we used to solve linear equations.

Addition property of inequality

For any algebraic expressions A, B, and C, if $A < B$, then

$$A + C < B + C$$

Multiplication property of inequality

For any algebraic expressions A, B, and C, if $A < B$, then

1. if $C > 0$ (positive), then

$$A \cdot C < B \cdot C$$

2. if $C < 0$ (negative), then

$$A \cdot C > B \cdot C$$

Concept

1. The same expression can be added to or subtracted from *both* members of an inequality and will not change the direction of the inequality symbol. We can multiply or divide *both* members of the inequality by the same positive expression and still maintain the direction of the inequality symbol.
2. We can multiply or divide *both* members of an inequality by the same *negative* expression provided that we *reverse* the direction of the inequality symbol.

Note The two properties are stated in terms of the is less than ($<$) symbol. These properties also apply for any of the other inequality symbols ($>$, $\leq$, or $\geq$).

Just as with equations, we can use the addition property to subtract the same expression from both members of an inequality. The multiplication property allows us to divide both members of an inequality by the same nonzero expression.

To demonstrate these operations, consider the inequality $8 < 12$.

1. If we add or subtract 4 in each member, we still have a true statement.

$8 < 12$	or	$8 < 12$	Original true statement
$8 + 4 < 12 + 4$		$8 - 4 < 12 - 4$	Add or subtract 4
$12 < 16$		$4 < 8$	New true statement

2. If we multiply or divide by 4 in each member, we still have a true statement.

$8 < 12$	or	$8 < 12$	Original true statement
$8 \cdot 4 < 12 \cdot 4$		$\frac{8}{4} < \frac{12}{4}$	Multiply or divide by 4
$32 < 48$		$2 < 3$	New true statement

3. But if we multiply or divide by -4 in each member, we must reverse the direction of the inequality to have a true statement.

$8 < 12$	or	$8 < 12$	Original true statement
$8(-4) > 12(-4)$		$\frac{8}{-4} > \frac{12}{-4}$	Multiply or divide by -4 and reverse direction of the inequality symbol
$-32 > -48$		$-2 > -3$	New true statement

Note When we reverse the direction of the inequality symbol, we say that we **reversed the sense or order** of the inequality.

To summarize our operations, we see that, with one exception, they are the same as the operations for linear equations. **Whenever we multiply or divide both members of an inequality by a negative number, we must reverse the direction of the inequality symbol.**

We shall now solve a linear inequality. The procedure for solving a linear inequality is the same four steps that we used to solve a linear equation.

Consider the inequality

$$3(4x - 1) \geq 5x + 3x + 5$$

Solving linear inequalities

Step 1 *We simplify* the inequality by carrying out the indicated multiplication in the left member and the addition in the right member.

$$\begin{aligned} 3(4x - 1) &\geq 5x + 3x + 5 \\ 12x - 3 &\geq 8x + 5 \qquad \text{Multiply and combine like terms} \end{aligned}$$

Step 2 *We want all terms containing the unknown, x, in one member of the inequality.* Therefore we subtract $8x$ from both members of the inequality.

$$\begin{aligned} 12x - 3 &\geq 8x + 5 \\ 12x - 8x - 3 &\geq 8x - 8x + 5 \qquad \text{Subtract } 8x \text{ from both members} \\ 4x - 3 &\geq 5 \qquad \text{Combine like terms} \end{aligned}$$

Note A negative coefficient of the unknown can be avoided if we form equivalent inequalities where the unknown appears only in the member of the inequality that has the greater coefficient of the unknown.

Step 3 *We want all terms not involving the unknown in the other member of the inequality.* Therefore we add 3 to both members of the inequality.

$$\begin{aligned} 4x - 3 &\geq 5 \\ 4x - 3 + 3 &\geq 5 + 3 \qquad \text{Add 3 to both members} \\ 4x &\geq 8 \end{aligned}$$

Step 4 *We form an equivalent inequality where the coefficient of the unknown is 1.* Hence we divide both members of the inequality by 4.

$$\begin{aligned} 4x &\geq 8 \\ \frac{4x}{4} &\geq \frac{8}{4} \qquad \text{Divide both members by 4} \\ x &\geq 2 \end{aligned}$$

The solution set is $\{x | x \geq 2\}$.

Note In step 4, we must be careful to observe whether we are multiplying or dividing by a positive or negative number so that we will form the correct inequality.

■ Example 2–5 C

Find the solution set of the following linear inequalities. Leave the answer in (1) set-builder notation, (2) graphical notation, and (3) interval notation.

1. $2(1 - 2x) \geq 4(2 - 3x) + 2x$

$$\begin{aligned} 2 - 4x &\geq 8 - 12x + 2x \qquad \text{Distributive property} \\ 2 - 4x &\geq 8 - 10x \qquad \text{Combine like terms} \\ 6x + 2 &\geq 8 \qquad \text{Add } 10x \text{ to both members} \\ 6x &\geq 6 \qquad \text{Subtract 2 from both members} \\ x &\geq 1 \qquad \text{Divide both members by 6} \end{aligned}$$

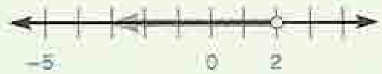
The solution set is $\{x | x \geq 1\}$

or $[1, +\infty)$

2. $4(3 - 2x) + 3x > 2$

$$\begin{aligned} 12 - 8x + 3x &> 2 && \text{Distributive property} \\ 12 - 5x &> 2 && \text{Combine like terms} \\ -5x &> -10 && \text{Subtract 12 from both members} \\ \frac{-5x}{-5} &< \frac{-10}{-5} && \text{Divide both members by } -5 \text{ and REVERSE THE DIRECTION OF THE INEQUALITY SYMBOL} \\ x &< 2 && \text{Simplify} \end{aligned}$$

The solution set is $\{x \mid x < 2\}$



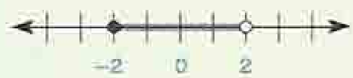
or $(-\infty,2)$

3. $-5 \leq 2x - 1 < 3$

When solving a compound inequality, the solution must be such that the unknown appears only in the middle member of the inequality. We can still use all of our properties, if we apply them to all *three* members. We must reverse the direction of *all* the inequality symbols when multiplying or dividing by a negative number.

$$\begin{aligned} -5 \leq 2x - 1 &< 3 \\ -5 + 1 \leq 2x - 1 + 1 &< 3 + 1 && \text{Add 1 to all three members} \\ -4 \leq 2x &< 4 && \text{Simplify} \\ \frac{-4}{2} \leq \frac{2x}{2} &< \frac{4}{2} && \text{Divide all three members by 2} \\ -2 \leq x &< 2 && \text{Simplify} \end{aligned}$$

The solution set is $\{x \mid -2 \leq x < 2\}$



or $[-2,2)$

4.

$$\begin{aligned} -12 < 8 - 4x &\leq 16 \\ -12 - 8 < 8 - 8 - 4x &\leq 16 - 8 && \text{Subtract 8 from all three members} \\ -20 < -4x &\leq 8 && \text{Simplify} \\ \frac{-20}{-4} > \frac{-4x}{-4} &\geq \frac{8}{-4} && \text{Divide all three members by } -4 \text{ and REVERSE THE DIRECTION OF ALL INEQUALITY SYMBOLS} \\ 5 > x &\geq -2 && \text{Simplify} \end{aligned}$$

The solution set is $\{x \mid -2 \leq x < 5\}$

or $[-2,5)$

Note When we state the answer in set-builder notation, it is customary to have compound inequalities stated with less than inequality symbols. This makes changing from one form of notation to another easier.

▶ ***Quick check*** Find the solution set of $3(2x + 1) \geq x + 2x + 7$. Give the answer in set-builder notation, interval notation, and graphical notation. ■

Problem solving

We are now ready to combine our ability to write an expression and our ability to solve an inequality and apply them to solve verbal problems. The guidelines for solving a linear inequality are the same as those for solving a linear equation in section 2–3. The following table shows a number of different ways an inequality symbol could be written with words.

Symbol	$<$	$\le$	$>$	$\ge$
In words	is less than is fewer than	is at most is no more than is no greater than is less than or equal to	is greater than is more than exceeds	is at least is no less than is no fewer than is greater than or equal to

Example 2–5 D

Solve the following word problems.

1. Five times a number is added to 6 and the result is no more than 41. Find all numbers that satisfy this condition.

   Let x represent the number. Then

five times a number	is added to	6	the result is **no more than**	41
$5x$	$+$	6	$\le$	41

   $5x + 6 \le 41$ Inequality
   $5x \le 35$ Subtract 6 from both members
   $x \le 7$ Divide both members by 5

   The solution set is $\{x \mid x \le 7\}$.

2. Three times a number plus 12 is at least -6 but less than 18. Find all numbers that satisfy these conditions.

   Let x represent the number. Then

-6 is at least	three times a number	plus	12	**is less than**	18
$-6 \le$	$3x$	$+$	12	$<$	18

   $-6 \le 3x + 12 < 18$ Compound inequality
   $-18 \le 3x < 6$ Subtract 12 from all three members
   $-6 \le x < 2$ Divide all three members by 3

   The solution set is $\{x \mid -6 \le x < 2\}$.

► ***Quick check*** Two times a number added to 7 is greater than 19. Find all numbers that satisfy this condition.

Mastery points

Can you

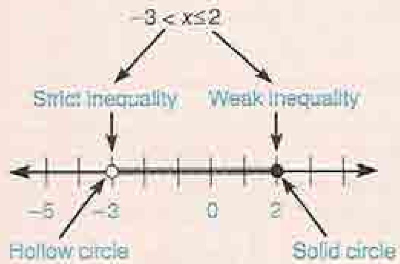
- Solve linear inequalities and compound inequalities?
- Represent the solution set of a linear inequality in set-builder notation, graphical notation, or interval notation?
- Solve word problems involving linear inequalities?

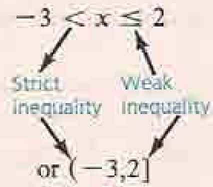
Exercise 2–5

Represent the following solution sets both graphically and with interval notation. See examples 2–5 A and B.

Example $\{x \mid -3 < x \le 2\}$

Solution





1. $\{x \mid -3 \le x \le 1\}$
2. $\{x \mid 0 < x < 4\}$
3. $\{x \mid -5 \le x < -1\}$
4. $\{x \mid x > -3\}$
5. $\{x \mid x \ge 4\}$
6. $\{x \mid x < 2\}$
7. $\{x \mid x \le -1\}$
8. $\{x \mid x \le 0\}$

Find the solution set of the following inequalities. Leave the answer in both set-builder notation and interval notation. See example 2–5 C.

Example $3(2x + 1) \ge x + 2x + 7$

Solution

$6x + 3 \ge 3x + 7$ — Distributive property and combine like terms

$3x + 3 \ge 7$ — Subtract $3x$ from both members

$3x \ge 4$ — Subtract 3 from both members

$x \ge \frac{4}{3}$ — Divide both members by 3

The solution set is $\left\{x \mid x \ge \frac{4}{3}\right\}$ or $\left[\frac{4}{3}, +\infty\right)$.

9. $2x > 18$
10. $3x < 12$
11. $\frac{2}{3}x \ge 8$
12. $\frac{3}{4}x \ge 9$
13. $-4x \le 20$
14. $-3x > 27$
15. $3x + 2x < x + 6$
16. $6x - 2x > 5x - 3$
17. $2x + (4x - 1) > 6 - x$
18. $3(2x + 1) < 9$
19. $4(2x - 5) \le 10x + 7$
20. $4 - 2(3x + 1) > 8x - 12$
21. $7 - 3(5x - 4) \le 12 - 9x$
22. $2(x - 4) - 14 \le 3(5 - 3x)$
23. $2(4x + 3) \ge 5 - 4(x - 1)$
24. $6(2x - 3) \le 9(x - 1) - 4$
25. $4(2 - x) + 7 > 3x - 3(x - 1)$
26. $4x - 4(x + 2) < 3(2 - x)$
27. $14 \ge 5(1 - x) + 2(x + 3)$
28. $7 < 4(1 - 2x) + 3(x - 4)$
29. $8.2x - 3.6x + 7.1 \ge 1.8x + 23.9$
30. $2.1(x - 6) < 0.4x + 7.8$
31. $4.3(x - 2) \ge 3.1x - 25.4$
32. $7.3(3 - x) < 4.9(2 - x) - 4.7$
33. $12.6(3x - 2) + 8.9 \le 8.9(2x - 4) - 0.7$
34. $-2 < 3x + 1 < 4$
35. $-3 < 3x - 5 < 4$
36. $-2 \le 4x + 2 \le 8$
37. $0 \le 7x - 2 \le 6$
38. $-5 < 4x + 5 \le 5$
39. $-4 \le 3x + 6 \le 6$

$\boxed{40.}$ $2 \le 1 - x \le 6$

41. $3 < 5 - 2x < 7$

42. $-2 < 4 - 3x \le 0$

43. $-5 \le 8 - 2x \le 0$

44. $0 \le 1 - 4x < 7$

45. $-4 \le 3 - 2x \le -1$

46. $-7 \le 4 - 2x \le -4$

47. $0 \le 2 - 3x \le 4$

48. $4 < 4 - 3x \le 6$

Write an inequality to represent the following statements. See example 2–5 D.

Example A student's score must be below 60 to fail the examination.

Solution Let x = the student's score. Then the inequality would be $x < 60$.

49. A student's score must be at least 90 to receive an A on the exam.

50. A student must score at least 75 on the final exam to pass the course.

51. The temperature today will not get above 42.

52. The temperature today will be at least 80.

53. A salesperson needs to sell at least 10 new cars to make a bonus.

$\boxed{54.}$ The temperature today will range from a low of 18 to a high of 41.

55. On a partly sunny day, there will be at least 96 minutes of sunlight but at most 384 minutes of sunlight.

56. The selling price P must be more than the cost c but less than twice the cost.

57. The selling price P must be at least one and one-half times the cost c but at most three times the cost.

Write an inequality using the given information and solve. See example 2–5 D.

Example Two times a number added to 7 is greater than 19. Find all numbers that satisfy this condition.

Solution Let x = the number. Then the inequality would be

$2x + 7 > 19$	Inequality
$2x > 12$	Subtract 7 from both members
$x > 6$	Divide both members by 2

The solution set is $\{x \mid x > 6\}$.

58. When 4 is added to three times a number, the result is at least 12. Find all numbers that satisfy this condition.

59. When 6 is subtracted from five times a number, the result is less than 17. Find all numbers that satisfy this condition.

60. Four times a number plus 6 is at least 21. Find all numbers that satisfy this condition.

61. If one-half of a number is added to 16, the result is greater than 24. Find all numbers that satisfy this condition.

62. Three times a number is subtracted from 11 and this result is less than 6. Find all numbers that satisfy this condition.

63. Two times a number is subtracted from 19 and this result is at most 8. Find all numbers that satisfy this condition.

$\boxed{64.}$ A student has scores of 7, 10, and 8 on three quizzes. What must she score on the fourth quiz to have an average of 8 or higher?

65. A student has scores of 66, 71, and 84 on three exams. If an average of 75 is required to pass the course, what is the minimum score he must have on the fourth test to pass?

66. Two times a number minus 6 is greater than 4 but less than 19. Find all numbers that satisfy these conditions.

67. Three times a number plus 2 is greater than 12 but less than 23. Find all numbers that satisfy these conditions.

68. The perimeter of a rectangle must be less than 100 feet. If the length is known to be 30 feet, find all numbers that the width could be. (*Note:* The width of a real rectangle must be a positive number.)

69. The perimeter of a square must be greater than 16 inches but less than 84 inches. Find all values of a side that satisfy these conditions. (*Hint:* The perimeter of a square is given by $P = 4s$, where s represents the length of a side.)

Review exercises

Write the value of the following numbers. See section 1–1.

1. $|-21|$
2. $|8|$
3. $-|-2|$

Write an algebraic expression for each of the following. See section 1–5.

4. 2 times y
5. 6 more than a
6. $\frac{1}{4}$ of a number
7. A number decreased by 12
8. A number increased by 7

2–6 ■ Inequalities involving absolute value

Absolute value inequalities of the form $|x| < a$

If the equals sign, $=$, in an absolute value equation is replaced with an inequality symbol, $<$, $>$, $\leq$, or $\geq$, the absolute value equation becomes an **absolute value inequality.** Consider the absolute value inequality

$$|x| < 2$$

This inequality states that the distance between x and the origin is less than 2 units.

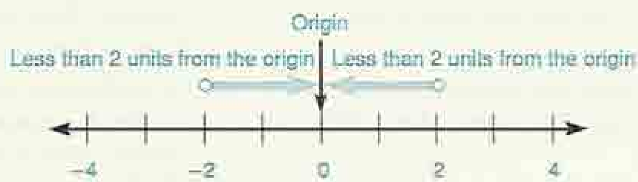


Figure 2–3

We see from figure 2–3 that all numbers between -2 and 2 satisfy the inequality $|x| < 2$. The solution set can be given in any one of the forms that we studied in section 2–5. That is, in set-builder notation, the solution set would be $\{x|-2 < x < 2\}$, in interval notation $(-2,2)$, and in graphical notation, as shown in figure 2–4.

Figure 2–4

We can generalize this observation in the following property.

$|x| < a$

For any real number x and $a > 0$,

$$|x| < a \text{ is equivalent to } -a < x < a$$

Concept

Given $|x| < a$, then x will be any real number between the opposite of a and a.

Note The property is stated in terms of the strict inequality $<$, but it is still true if we replace the strict inequality symbol with the weak inequality symbol $\leq$. That is, $|x| \leq a$ is equivalent to $-a \leq x \leq a$.

Solving absolute value inequalities of the type $|x| < a$

1. Isolate the absolute value in one member of the inequality.
2. Write the equivalent inequality (a three-member compound inequality).
3. Solve the inequality.

Solutions for the absolute value inequality $|x| < a$

If a is positive, the inequality is $-a < x < a$	If a is zero or negative, there is no solution.*

Example 2–6 A

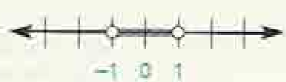
Find the solution set of the following absolute value inequalities and leave the answer in (1) set-builder notation, (2) interval notation, and (3) graphical notation.

1. $|x| - 3 < -2$

$|x| < 1$ — Isolate the absolute value by adding 3

$-1 < x < 1$ — Write the equivalent compound inequality

The solution set is $\{x|-1 < x < 1\}$, or $(-1,1)$, or



2. $|x + 3| \leq 6$

$-6 \leq x + 3 \leq 6$ — Write the equivalent compound inequality

$-9 \leq x \leq 3$ — Subtract 3 from all three members

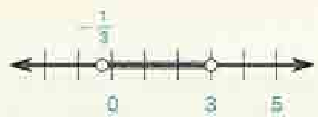
The solution set is $\{x|-9 \leq x \leq 3\}$, or $[-9,3]$, or

*If a is negative or zero, then there is no solution since $|x|$ is always greater than or equal to zero and cannot be less than zero or any negative value that a is representing.

3.

$$
\begin{aligned}
|3x - 4| &< 5 \\
-5 < 3x - 4 &< 5 && \text{Write the equivalent compound inequality} \\
-1 < 3x &< 9 && \text{Add 4 to all three members} \\
-\frac{1}{3} < x &< 3 && \text{Divide all three members by 3}
\end{aligned}
$$

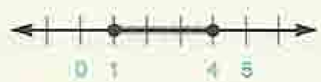
The solution set is $\left\{x \middle| -\frac{1}{3} < x < 3\right\}$, or $\left(-\frac{1}{3}, 3\right)$, or



4.

$$
\begin{aligned}
|5 - 2x| &\le 3 \\
-3 \le 5 - 2x &\le 3 && \text{Write the equivalent compound inequality} \\
-8 \le -2x &\le -2 && \text{Subtract 5 from all three members} \\
\frac{-8}{-2} \ge \frac{-2x}{-2} &\ge \frac{-2}{-2} && \text{Divide by } -2 \text{ and } \textbf{REVERSE the direction of all the inequality symbols} \\
4 \ge x &\ge 1 && \text{Simplify}
\end{aligned}
$$

The solution set is $\{x | 1 \le x \le 4\}$, or $[1,4]$, or



5. $|7x - 4| < -5$

The solution set is $\emptyset$. Since the absolute value cannot be less than zero, there is no solution

Absolute value inequalities of the form $|x| > a$

We have only considered those absolute value inequalities that involve "is less than," $<$, or "is less than or equal to," $\le$. Consider the absolute value inequality

$$|x| > 2$$

This inequality states that the distance between x and the origin is more than 2 units.

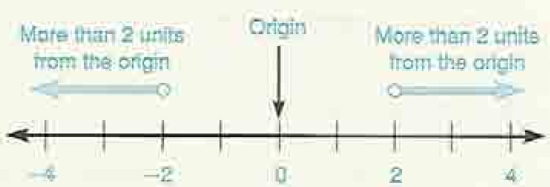


Figure 2–5

We see from figure 2–5 that all numbers greater than 2 or less than -2 satisfy the inequality $|x| > 2$. The solution set is the union of the two intervals, $x > 2$ or $x < -2$, and is given by

$$\{x | x < -2 \text{ or } x > 2\} \text{ or } (-\infty, -2) \cup (2, +\infty)$$

or as shown in figure 2–6.

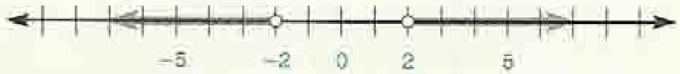


Figure 2–6

We can generalize this observation in the following property.

$|x| > a$

For any real number x and $a > 0$,

$$|x| > a \text{ is equivalent to } x < -a \text{ or } x > a$$

Concept

Given $|x| > a$, then x will be any real number less than the opposite of a or greater than a.

Note The property is stated in terms of the strict inequality $>$, but it is still true if we replace the strict inequality symbol with the weak inequality symbol. That is, $|x| \geq a$ is equivalent to $x \leq -a$ or $x \geq a$.

Solving absolute value inequalities of the form $|x| > a$

1. Isolate the absolute value in one member of the inequality.
2. Write the equivalent inequalities (a pair of inequalities connected with an "or").
3. Solve the inequalities.

Solutions for the absolute value inequality $|x| > a$

If a is positive, the inequalities are	If a is negative, then the solution set is the set of all real numbers.*
$x < -a$ or $x > a$	R

Example 2–6 B

Find the solution set of the following absolute value inequalities and leave the answer in (1) set-builder notation, (2) interval notation, and (3) graphical notation.

1. $|x + 2| > 2$

$x + 2 < -2$ or $x + 2 > 2$ — Write the equivalent inequalities

$x < -4 \qquad x > 0$ — Subtract 2 from both members of both inequalities

The solution set is $\{x \mid x < -4 \text{ or } x > 0\}$, or $(-\infty,-4) \cup (0,+\infty)$, or

*Any real number will be a solution since the absolute value of any real number is greater than or equal to zero and zero is greater than any negative value that a is representing.

2. $|3x - 4| \geq 2$

$$
\begin{array}{lcll}
3x - 4 \leq -2 & \text{or} & 3x - 4 \geq 2 & \text{Write the equivalent inequalities} \\
3x \leq 2 & & 3x \geq 6 & \text{Add 4 to both members of both inequalities} \\
x \leq \dfrac{2}{3} & & x \geq 2 & \text{Divide all members by 3}
\end{array}
$$

The solution set is $\left\{x \middle| x \leq \dfrac{2}{3} \text{ or } x \geq 2\right\}$, or $\left(-\infty, \dfrac{2}{3}\right] \cup [2, +\infty)$, or

3. $|4x + 3| - 4 > 7$

$$
\begin{array}{lcll}
|4x + 3| > 11 & & & \text{Isolate the absolute value by adding 4} \\
4x + 3 < -11 & \text{or} & 4x + 3 > 11 & \text{Write the equivalent inequalities} \\
4x < -14 & & 4x > 8 & \text{Subtract 3 from all members} \\
x < -\dfrac{7}{2} & & x > 2 & \text{Divide all members by 4}
\end{array}
$$

The solution set is $\left\{x \middle| x < -\dfrac{7}{2} \text{ or } x > 2\right\}$, or $\left(-\infty, -\dfrac{7}{2}\right) \cup (2, +\infty)$, or

4. $|4x - 5| + 8 \geq 3$

$$
\begin{array}{ll}
|4x - 5| \geq -5 & \text{Isolate the absolute value by subtracting 8}
\end{array}
$$

The solution set is $\{x | x \in R\}$, or $(-\infty, +\infty)$, or

Since this is true for all values of x, the solution set is all real numbers

Note A common error in solving absolute value inequalities is to apply the wrong procedure. Once you have the absolute value isolated in one member of the inequality, identify the type of inequality symbol and apply the appropriate property.

▶ ***Quick check*** Find the solution set of $|5x + 4| - 6 \geq -3$. Give the answer in set-builder notation, interval notation, and graphical notation. ■

Mastery points

Can you

- Solve absolute value inequalities?

Exercise 2–6

Determine whether or not the given value is an element of the solution set of the absolute value inequality. Use $|3x + 2| \geq 5$ for exercises 1–4 and $|2x - 3| < 7$ for exercises 5–8.

Example Is 6 an element of the solution set of the absolute value inequality $|2x - 3| < 7$?

Solution

$$|2(6) - 3| < 7 \quad \text{Substitute}$$
$$|12 - 3| < 7$$
$$|9| < 7$$
$$9 < 7 \quad \text{(False)}$$

Therefore 6 is not an element of the solution set of $|2x - 3| < 7$.

1. 0 **2.** -1 **3.** 1 **4.** -3 **5.** 0 **6.** 4 **7.** 5 **8.** 10

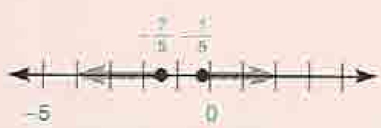
Solve the following absolute value inequalities. For exercises 9–18, leave the answer in set-builder notation, interval notation, and graphical notation. For exercises 19–47, leave the answer in both set-builder and interval notation. See examples 2–6 A and B.

Example $|5x + 4| - 6 \geq -3$

Solution

$$|5x + 4| \geq 3 \quad \text{Isolate the absolute value by adding 6}$$
$$5x + 4 \leq -3 \quad \text{or} \quad 5x + 4 \geq 3 \quad \text{Write the equivalent inequalities}$$
$$5x \leq -7 \qquad 5x \geq -1 \quad \text{Subtract 4 from all members}$$
$$x \leq -\frac{7}{5} \qquad x \geq -\frac{1}{5} \quad \text{Divide all members by 5}$$

The solution set is $\left\{x \mid x \leq -\frac{7}{5} \text{ or } x \geq -\frac{1}{5}\right\}$, or $\left(-\infty, -\frac{7}{5}\right] \cup \left[-\frac{1}{5}, +\infty\right)$, or



Graphical notation

9. $|x| < 4$
10. $|x| \leq 3$
11. $|x| \geq 2$
12. $|x| > 4$
13. $|x| - 2 \leq 1$
14. $|x| - 1 < 2$
15. $|x| + 3 > 8$
16. $|x| + 5 \geq 7$
17. $|3x - 4| < 6$
18. $|2x + 5| \leq 4$
19. $|5x - 3| \geq 7$
20. $|4x - 5| > 9$
21. $|3x - 4| \leq 13$
22. $|2x + 7| > 11$
23. $|5x + 7| < 12$
24. $|1 - 2x| \leq 5$
25. $|4 - 3x| \leq 13$
26. $|6 - 3x| < 12$
27. $|5 - 2x| < 15$
28. $|4x - 9| < 0$
29. $|3x + 6| \leq -1$
30. $|5 - 8x| < -3$
31. $|4x - 6| \geq -2$
32. $|3x + 7| > -9$
33. $|7x - 4| + 5 < 7$
34. $|3x - 11| + 6 < 9$
35. $|1 - 3x| - 4 \geq 3$
36. $|2 - 7x| - 3 \geq 4$
37. $|3x - 4| + 5 < 4$
38. $|7x - 8| + 6 \leq 3$
39. $|4x - 9| + 6 \geq 4$
40. $4 + |3x + 1| > 6$
41. $8 + |5x - 3| > 10$

42. $4 + |3 - 2x| \geq 7$ **43.** $|4.8x - 18.42| > 11.34$ **44.** $|3.2x - 12.2| > 13.4$

45. $|8.2x - 6.15| \leq 10.25$ **46.** $|10.8x - 10.8| \leq 5.4$ **47.** $|2.1x - 6.3| < 8.4$

Write an absolute value equation or inequality for the following statements and solve for the unknown. Leave the solution of inequalities in both set-builder and interval notation.

Example The absolute value of a number is more than 3.

Solution If we let x represent the unknown number, then the absolute value inequality would be $|x| > 3$.

$x < -3$ or $x > 3$ Write the equivalent inequalities

The solution set is $\{x|x < -3 \text{ or } x > 3\}$ or $(-\infty, -3) \cup (3, +\infty)$.

48. The absolute value of a number is equal to 8.

49. The absolute value of a number is equal to 12.

50. The absolute value of twice a number is equal to 10.

51. The absolute value of $\frac{1}{2}$ a number is equal to 7.

52. If 6 is subtracted from three times a number, the absolute value of the result is equal to 11.

53. If 3 is added to four times a number, the absolute value of the result is equal to 19.

54. The absolute value of a number is less than 4.

55. The absolute value of a number is at most 6.

56. The absolute value of a number is at least 8.

57. The absolute value of twice a number is less than 14.

58. If three times a number is decreased by 4, the absolute value of the result is at least 11.

59. If twice a number is increased by 5, the absolute value of the result is more than 15.

60. If $\frac{1}{2}$ of a number is diminished by 6, the absolute value of the result is at most 14.

61. If $\frac{1}{4}$ of a number is decreased by 8, the absolute value of the result is less than 12.

Review exercises

Perform the indicated operations. See section 1–2.

1. -4^2 **2.** $(-4)^2$ **3.** -2^4 **4.** $(-2)^4$

Write an algebraic expression for each of the following. See section 1–5.

5. x raised to the fifth power

6. A number cubed

7. A number squared

8. The product of x and y

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Chapter 2 lead-in problem

Earl invests a total of \$10,000. He invests part in Collins Feline Fanciers that pays a 9% dividend per year and the rest in Grutz Shipyards that pays an 8% dividend per year. If Earl receives \$870 per year from his investments, how much did he invest with each company?

Solution

Let x represent the amount invested at 9%, then $10{,}000 - x$ will represent the amount invested at 8%. We can use a table to summarize the information in the problem.

	Investment earning 9%	Investment earning 8%	Total
Amount invested	x	$10{,}000 - x$	10,000
Dividend received	$(0.09)x$	$(0.08)(10{,}000 - x)$	870

We get the equation for the problem from the bottom row of the table.

$$\underbrace{(0.09)x}_{\text{amount of dividend at 9\%}} \underbrace{+}_{\text{total}} \underbrace{(0.08)(10{,}000 - x)}_{\text{amount of dividend at 8\%}} \underbrace{=}_{\text{was}} \underbrace{870}_{\text{total dividend}}$$

Solving for x,

$$\begin{aligned} (0.09)x + (0.08)(10{,}000 - x) &= 870 && \text{Equation} \\ (0.09)x + 800 - (0.08)x &= 870 && \text{Distributive property} \\ (0.01)x + 800 &= 870 && \text{Combine like terms} \\ (0.01)x &= 70 && \text{Subtract 800} \\ x &= 7{,}000 && \text{Divide by 0.01} \end{aligned}$$

and $10{,}000 - x = 10{,}000 - 7{,}000 = 3{,}000$ Substitute to get other investment

Hence the amount invested at 9% in Collins Feline Fanciers (x) was \$7,000 and the amount invested at 8% in Grutz Shipyards ($10{,}000 - x$) was \$3,000.

Chapter 2 summary

1. A **mathematical statement** is a sentence that can be labeled true or false.
2. An **equation** is a statement of equality.
3. A replacement value for the variable that forms a true statement (satisfies that equation) is called a **root,** or **solution,** of that equation.
4. The set of all those values for the variable that causes the equation to be a true statement is called the **solution set** of the equation.
5. An equation that is true for some values of the variable and false for other values of the variable is called a **conditional equation.**
6. An equation that is true for every permissible value of the variable is called an **identical equation,** or **identity.**
7. In a **first-degree conditional equation** in one variable, also called a **linear equation,** the exponent of the unknown is 1 and the solution set will contain at most one root.

8. The **addition property of equality** enables us to add to or subtract the same amount from each member of an equation and the result will be an equivalent equation.
9. The **multiplication property of equality** enables us to multiply or divide both members of an equation by the same nonzero number and the result will be an equivalent equation.
10. Whenever we multiply or divide both members of an inequality by a negative number, we must **reverse the direction of the inequality symbol.**
11. In **interval notation** we use a parenthesis, (or), to represent that the endpoint is *not* part of the interval and a bracket, [or], to represent that the endpoint is part of the interval.
12. If $|x| = a$ and $a \geq 0$, then $x = a$ or $x = -a$.
13. If $|x| < a$ and $a > 0$, then $-a < x < a$.
14. If $|x| > a$ and $a > 0$, then $x < -a$ or $x > a$.

Chapter 2 error analysis

1. Solving fractional equations
   Example: $\frac{3}{4}x + 2 = \frac{1}{2}x$
   $4 \cdot \frac{3}{4}x + 2 = 4 \cdot \frac{1}{2}x$
   $3x + 2 = 2x$
   $x = -2 \qquad \{-2\}$
   Correct answer: $\{-8\}$
   What error was made? (*see page 55*)
2. Solving literal equations
   Example: Solve $3x + 2y - 1 = 5x + 4y$ for x.
   $3x + 2y - 1 = 5x + 4y$
   $3x - 5x = 4y - 2y + 1$
   $-2x = 2y + 1$
   $x = -y + 1$
   Correct answer: $x = -y - \frac{1}{2}$
   What error was made? (*see page 60*)
3. Solving an absolute value equation
   Example: The solution set of the equation $|x| = -2$ is $\{2\}$.
   Correct answer: $\emptyset$
   What error was made? (*see page 73*)
4. Solving an absolute value equation
   Example: Find the solution set of
   $|2x - 1| = 9$
   $2x - 1 = 9$
   $2x = 10$
   $x = 5 \qquad \{5\}$
   Correct answer: The solution set is $\{-4,5\}$.
   What error was made? (*see page 73*)
5. Solution sets of linear inequalities
   Example: The graph of $\{x|x \geq 2\}$ is

   What error was made? (*see page 77*)
6. Multiplying an inequality by a negative number
   Example: If $-2 \leq -\frac{1}{3}x < 4$, then
   $-3 \cdot -2 \leq -3 \cdot -\frac{1}{3}x < -3 \cdot 4$
   Correct answer: $-3 \cdot -2 \geq -3 \cdot -\frac{1}{3}x > -3 \cdot 4$
   What error was made? (*see page 80*)
7. Solving linear inequalities
   Example: Find the solution set of
   $5 - 4x \geq 9$
   $-4x \geq 4$
   $x \geq -1 \qquad \{x|x \geq -1\}$
   Correct answer: $\{x|x \leq -1\}$
   What error was made? (*see page 82*)
8. Solving absolute value inequalities
   Example: Find the solution set of
   $|x - 2| < 5$
   $x - 2 < 5$
   $x < 7$
   $\{x|x < 7\}$
   Correct answer: The solution set is $\{x|-3 < x < 7\}$.
   What error was made? (*see page 87*)
9. Solving absolute value inequalities
   Example: Find the solution set of $|x + 6| \geq 5$.
   If $|x + 6| \geq 5$, then $-5 \leq x + 6 \leq 5$
   $-11 \leq x \leq -1$
   $\{x|-11 \leq x \leq -1\}$.
   Correct answer: $\{x|x \leq -11 \text{ or } x \geq -1\}$
   What error was made? (*see page 89*)
10. Removing grouping symbols
   Example:
   $(3a^2 + 2a - 1) + [4a^2 - (a + 1)]$
   $= 3a^2 + 2a - 1 + 4a^2 - a + 1$
   $= 7a^2 + a$
   Correct answer:
   $(3a^2 + 2a - 1) + [4a^2 - (a + 1)] = 7a^2 + a - 2$
   What error was made? (*see page 42*)

Chapter 2 critical thinking

Add any two consecutive integers. Add 7 to that sum and divide the result by 2. If you subtract the original number from this quotient, the result will always be 4. Why is this true?

Chapter 2 review

[2–1]

Find the solution set of the following linear equations.

1. $4x = 32$ **2.** $x + 11 = 17$ **3.** $\frac{a}{7} = 4$

4. $\frac{5b}{2} = 10$ **5.** $7a - 2 = 13$ **6.** $2(3z - 4) = 12$

7. $4(3 - 2x) + 3 = 4x - 5$ **8.** $4(3a + 2) - 2a = 5(a - 1)$ **9.** $7.8a - 16.9 = 4.3a + 14.6$

[2–2]

Solve the following formulas or literal equations for the specified variable. Assume that no denominator is equal to zero.

10. $v = \ell wh; w$ **11.** $v = k + gt; t$ **12.** $D = dq + R; d$

13. $v = r^2(a - b); b$ **14.** $2s = 2vt - gt^2; v$ **15.** $\ell = a + (n - 1)d; n$

16. $3x - y = 5x - 4y; x$

[2–3]

Write an equation for the problem and solve for the unknown quantities.

17. If three times a number is increased by 15 and the answer is 51, what is the number?

18. One-third of a number is 6 less than one-half of the number. Find the number.

19. The sum of three numbers is 27. The second number is three times the first and the third number is 6 less than the first. Find the three numbers.

20. The length of a rectangle is 8 feet more than twice the width. The perimeter is 82 feet. Find the dimensions.

21. Mary Ann has \$24,000, part of which she invests at 10% interest and the rest at 8%. If her income for one year from the two investments was \$2,220, how much did she invest at each rate?

22. A solution that is 42% hydrochloric acid is to be mixed with a solution that is 12% hydrochloric acid to obtain 100 centiliters of solution that is 24% hydrochloric acid. How many centiliters of each solution should be used in the mixture?

[2–4]

Find the solution set of the following absolute value equations.

23. $|x| - 4 = 11$ **24.** $|3a + 5| = 12$ **25.** $|7 - 2x| = 10$

26. $|4c - 6| + 12 = 18$ **27.** $|3a + 4| = |2a - 3|$ **28.** $|4y + 6| = |2y - 5|$

[2–5]

Find the solution set of the following linear inequalities. Leave the answer in both set-builder notation and interval notation.

29. $5x \leq 30$ **30.** $\frac{3}{4}x > 12$ **31.** $-2x < 9$

32. $2(3x - 4) \leq 1 - 2x$ **33.** $10 - 2(3x - 4) > 9 - 12x$ **34.** $5(2x - 3) \leq 7(x + 1) + 3$

35. $-4 < 2x + 3 < 5$ **36.** $0 \leq 5x + 4 \leq 4$ **37.** $5 < 3 - x < 8$

38. $-6 \leq 4 - 3x < 2$ **39.** $6 < 6 - 4x \leq 10$

Write an inequality using the given information and solve.

40. When 5 is subtracted from four times a number, the result is at least 19. Find all numbers that satisfy this condition.

41. Three times a number plus 7 is greater than 22 but less than 34. Find all numbers that satisfy these conditions.

Solve the following absolute value inequalities. Leave the answer in both set-builder and interval notation.

42. $|x| - 4 \geq 6$
43. $|2x + 5| < 6$
44. $|5x - 1| \leq 7$
45. $|4x + 7| > 9$
46. $|1 - 3x| \geq 5$
47. $|3 - 4x| < 12$
48. $|5x + 1| > -3$
49. $6 + |1 - 2x| < 10$
50. $|4x + 5| - 5 \geq 8$
51. $|6x + 3| < -4$

Chapter 2 cumulative test

Determine if the following statements are true or false.

[1–1] **1.** $\frac{1}{2} \in J$
[1–1] **2.** $5 \subseteq R$
[1–1] **3.** $0 \in W$
[1–1] **4.** $Q \cup H = R$
[1–1] **5.** $|-4| < |-6|$
[1–1] **6.** $\{4\} \subseteq \{1,2,3,4\}$
[1–1] **7.** $Q \cap H = \emptyset$

Perform the indicated operations if possible and simplify.

[1–2] **8.** $(-4)(+3)(-2)$
[1–2] **9.** $(-8) - (-7)$
[1–2] **10.** $(-6)(-2)$
[1–2] **11.** $\frac{(-5) + 5}{(-2)}$
[1–2] **12.** $25 - (5 - 11) + 6$
[1–2] **13.** $\frac{(-4)(-9)}{(2)(-3)}$
[1–2] **14.** -7^2

Identify which property of real numbers is being used.

[1–3] **15.** $a(b + c) = (b + c)a$
[1–3] **16.** $(xy)z = z(xy)$

Form the following sets.

[1–1] **17.** $\{1,2,3\} \cup \{2,3,4,5\}$
[1–1] **18.** $\{8,10,11\} \cup \{4,6,9\}$
[1–1] **19.** $\{10,11,12,13\} \cap \{10,12\}$

Solve the following literal equations for the specified variable and find the solution set for the linear equations and linear inequalities. Assume that no denominator is equal to zero.

[2–1] **20.** $6x + 5 = 2x + 12$
[2–2] **21.** $M = -P(\ell - x);\ P$
[2–2] **22.** $P = n(P_2 - P_1) - c;\ P_2$
[2–5] **23.** $-3x \leq 15$
[2–6] **24.** $|4x + 6| \geq 5$
[2–1] **25.** $5(2 - 3x) + 4 = 2x - 7$
[2–5] **26.** $7(3 - 4x) + 6 \leq 6(2 - 4x)$
[2–4] **27.** $|2a + 3| = |3a + 2|$
[2–1] **28.** $6(3a - 5) - 4a = 7(a - 2)$
[2–1] **29.** $3(2x - 4) + 6 = 6x + 8$
[2–6] **30.** $|4 - 3x| \leq 7$

[2–3] **31.** The sum of three numbers is 72. The first number is twice the second number and the third number is three times the first number. Find the three numbers.

[2–3] **32.** Harold has \$40,000, part of which he invests at 11% and the rest at 8%. If his income for one year from the 11% investment is \$1,740 more than that from the 8% investment, how much did he invest at each rate?

60. let n = the number; $\frac{n+4}{8}$ **61.** $13y^2 - 9y$
62. $14x^2y + 5x^2y^2 - xy^2$ **63.** $8x + 17$ **64.** $4a - 4$
65. $5x - 2$ **66.** $11c - 6$ **67.** $2x^2 - 5x + 11$ **68.** $8a + 4$
69. $-10x$ **70.** $-2a^2 - 3b$

Chapter 1 test

1. false **2.** true **3.** true **4.** $\{2,4,6,8\}$ **5.** $\{4,6\}$ **6.** $\{2,4\}$
7. -14 **8.** -6 **9.** 9 **10.** $-x - 5$ **11.** -9 **12.** 0
13. $7a$ **14.** 10 **15.** -3 **16.** 16 **17.** 8 **18.** $\frac{5}{12}$ **19.** 0
20. $4a + 5b$ **21.** 15 **22.** 5 **23.** 6 **24.** let x = the number; $x - 3$ **25.** let x = the number; $\frac{x+5}{8}$

Chapter 2

Exercise 2–1

Answers to odd-numbered problems

1. $\{3\}$ **3.** $\left\{\frac{5}{2}\right\}$ **5.** $\{6\}$ **7.** $\{3\}$ **9.** $\{12\}$ **11.** $\left\{\frac{32}{3}\right\}$
13. $\{12\}$ **15.** $\left\{\frac{27}{2}\right\}$ **17.** $\{3\}$ **19.** $\{-1\}$ **21.** $\left\{\frac{5}{3}\right\}$
23. $\left\{\frac{1}{3}\right\}$ **25.** $\left\{\frac{15}{14}\right\}$ **27.** $\left\{\frac{5}{3}\right\}$ **29.** $\emptyset$ **31.** $\left\{-\frac{7}{2}\right\}$
33. $\{8\}$ **35.** $\{24\}$ **37.** $\left\{\frac{9}{2}\right\}$ **39.** $\{2\}$ **41.** $\left\{\frac{31}{3}\right\}$ **43.** $\left\{\frac{-6}{7}\right\}$
45. $\{6\}$ **47.** $\{-8.4\}$ **49.** $\{8\}$ **51.** $\emptyset$ **53.** $\ell = 8$
55. $t = 4$ years **57.** $115c$ **59.** $\frac{18}{h}$ **61.** $50h + 25q + 10d + 5n$
63. $3n, 3n - 8$ **65.** $d + 464 - 5m$ **67.** $w + 1$ **69.** $j + 2$
71. $2d + 500$ **73.** $59c + 115b$ **75.** $20(w + 11)$

Solutions to trial exercise problems

18.
$$\begin{aligned} 4x + 5 &= 5 \\ 4x &= 0 \\ x &= 0 \\ &\{0\} \end{aligned}$$

44.
$$\begin{aligned} 5.6z - 22.15 &= 24.33 \\ 5.6z &= 46.48 \\ z &= 8.3 \\ &\{8.3\} \end{aligned}$$

50.
$$\begin{aligned} 3(2x - 1) &= 6x + 7 \\ 6x - 3 &= 6x + 7 \\ -3 &= 7 \text{ (false)} \\ &\emptyset \end{aligned}$$

Review exercises

1. 264 **2.** 600 **3.** 220 **4.** 38.75 **5.** 256 **6.** 38

Exercise 2–2

Answers to odd-numbered problems

1. $R = 9$ **3.** $P = 3{,}000$ **5.** $n = 20$ **7.** $V_1 = 51$ **9.** $t = \frac{I}{pr}$
11. $m = \frac{E}{c^2}$ **13.** $m = \frac{F}{a}$ **15.** $b = \frac{A}{h}$ **17.** $R = \frac{W}{I^2}$
19. $k = V - gt$ **21.** $g = \frac{V - k}{t}$ **23.** $q = \frac{D - R}{d}$
25. $\ell = \frac{px - m}{p}$ **27.** $W = R + 2bc + b^2$ **29.** $a = \frac{V + br^2}{r^2}$
31. $d = \frac{2S - 2an}{n^2 - n}$ **33.** $g = \frac{2Vt - 2S}{t^2}$ **35.** $d = \frac{\ell - a}{n - 1}$
37. $x = \frac{12 - 3y}{2}$ **39.** $y = \frac{-3x}{7}$ **41.** $x = \frac{5y + 6}{10}$
43. $y = \frac{4x - 3}{a}$ **45.** $y = \frac{ax + 3a + 4b}{b}$ **47.** $v = \frac{2s - gt^2}{2t}$
49. $P_1 = \frac{nP_2 - P - c}{n}$

Solution to trial exercise problem

26.
$$\begin{aligned} R &= W - b(2c + b);\ c \\ R &= W - 2bc - b^2 \\ 2bc + R &= W - b^2 \\ 2bc &= W - b^2 - R \\ c &= \frac{W - b^2 - R}{2b} \end{aligned}$$

Review exercises

1. $3x$ **2.** $6(a + 7)$ **3.** $\frac{y - 2}{4}$ **4.** let n = the number; $5n$
5. let n = the number; $n - 12$ **6.** let n = the number; $\frac{n}{8} - 9$

Exercise 2–3

Answers to odd-numbered problems

1. 40,48 **3.** 6,48 **5.** 21,26 **7.** 18 **9.** 84 **11.** -26
13. 17 **15.** 23,58 **17.** 13 **19.** 26,38 **21.** 47,79 **23.** 5,19
25. 22,23,24 **27.** 14,16,18 **29.** $-23, -21, -19$ **31.** 9,36
33. 24 **35.** 29,40 **37.** 21,22 **39.** 8,10,12 **41.** 11,33,5
43. 8,15 **45.** \$10,000 at 8%; \$5,000 at 6% **47.** \$13,000 at 10%; \$13,000 at 12% **49.** \$5,000 at 10%; \$7,000 at 12%
51. \$12,285.71 at 5%; \$5,714.29 at 9% **53.** \$4,000
55. \$14,000 at 14%; \$12,000 at 10% **57.** \$10,000 at 14%; \$8,000 at 9% **59.** \$19,000 at 12%; \$15,000 at 21%
61. ℓ = 28 feet, w = 23 feet **63.** ℓ = 48 feet, w = 16 feet
65. 14 cm, 7 cm, 17 cm **67.** 40 cm³ of 10% solution, 80 cm³ of 4% solution **69.** 3 ounces of 60% gold, 9 ounces of 80% gold **71.** 100 3-grain capsules, 100 2-grain capsules
73. 10 liters of 60% solution, 20 liters of 30% solution

Solutions to trial exercise problems

9. Let n = the number.

a number is divided by 4	increased by	6	is	27
$\frac{n}{4}$	$+$	6	$=$	27

$\frac{n}{4} + 6 = 27$	Original equation
$4\left(\frac{n}{4} + 6\right) = 4 \cdot 27$	Multiply by 4 to clear the fraction
$n + 24 = 108$	Distribute the multiplication in the left member
$n = 84$	Subtract 24

The number is 84.

52. x = the amount invested at 10%.

\$5,000 at 8%	more invested at 10%	will be	total at 9%
5,000(0.08)	$+ x(0.10)$	=	$(5{,}000 + x)(0.09)$

Solving for x: $5{,}000(0.08) + 0.10x = (5{,}000 + x)(0.09)$

$$400 + 0.10x = 450 + 0.09x$$
$$400 + 0.01x = 450$$
$$0.01x = 50$$
$$x = 5{,}000$$

Therefore \$5,000

	Investment at 8%	Investment at 10%	Total investment at 9%
Amount invested	5,000	x	$5{,}000 + x$
Interest received	5,000(0.08)	$x(0.10)$	$(5{,}000 + x)(0.09)$

Review exercises

1. 12 **2.** 0 **3.** $\frac{3}{4}$ **4.** 6 **5.** $\left\{\frac{20}{3}\right\}$ **6.** $\{-6\}$ **7.** $\{5\}$ **8.** $\{-7\}$

Exercise 2-4

Answers to odd-numbered problems

1. true **3.** false **5.** false **7.** true **9.** $\{9,-9\}$ **11.** $\{4,-4\}$ **13.** $\{4,-4\}$ **15.** $\{-12,12\}$ **17.** $\{2,-10\}$ **19.** $\{1,5\}$ **21.** $\left\{\frac{-4}{3},4\right\}$ **23.** $\{-8,1\}$ **25.** $\emptyset$ **27.** $\emptyset$ **29.** $\left\{-\frac{3}{4},\frac{9}{4}\right\}$ **31.** $\left\{-\frac{13}{5},\frac{9}{5}\right\}$ **33.** $\{-1,7\}$ **35.** $\{3,9\}$ **37.** $\{-24,4\}$ **39.** $\left\{-\frac{16}{3},\frac{32}{3}\right\}$ **41.** $\{3,15\}$ **43.** $\left\{-\frac{7}{3},\frac{17}{3}\right\}$ **45.** $\left\{0,\frac{3}{2}\right\}$ **47.** $\left\{-5,\frac{1}{2}\right\}$ **49.** $\left\{-\frac{3}{2},-\frac{1}{2}\right\}$ **51.** $\left\{-\frac{9}{2},-\frac{1}{2}\right\}$ **53.** $\left\{-\frac{2}{3}\right\}$ **55.** $\left\{-\frac{1}{2}\right\}$ **57.** $\{-6,6\}$ **59.** $\{-4,4\}$ **61.** $\{-6,22\}$ **63.** $\{-22,8\}$ **65.** $\left\{-\frac{4}{3},4\right\}$ **67.** $\{-21,63\}$

Solutions to trial exercise problems

25. $|5x - 3| = -4$

$\emptyset$ since the absolute value can only be greater than or equal to zero.

43. $|5 - 3x| - 4 = 8$

$$|5 - 3x| = 12$$

$5 - 3x = 12$ or $5 - 3x = -12$

$-3x = 7$ $\quad$ $-3x = -17$

$x = -\frac{7}{3}$ $\quad$ $x = \frac{17}{3}$

$$\left\{-\frac{7}{3},\frac{17}{3}\right\}$$

47. $|3x - 7| = |5x + 3|$

$3x - 7 = 5x + 3$ or $3x - 7 = -(5x + 3)$

$-7 = 2x + 3$ $\quad$ $3x - 7 = -5x - 3$

$-10 = 2x$ $\quad$ $8x - 7 = -3$

$-5 = x$ $\quad$ $8x = 4$

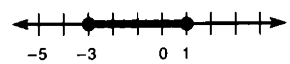
$\left\{-5,\frac{1}{2}\right\}$ $\quad$ $x = \frac{4}{8} = \frac{1}{2}$

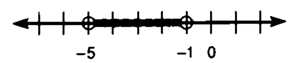
Review exercises

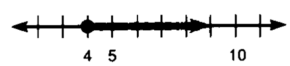
1. $x + 2$ **2.** $y - 6$ **3.** $a - 4$ **4.** let x = the number; $\frac{x}{5}$ **5.** let x = the number; $\frac{1}{3}x$ **6.** let x = the number; $\frac{x-2}{8}$

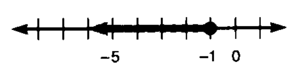
Exercise 2-5

Answers to odd-numbered problems

1. $[-3,1]$ 

3. $(-5,-1)$ 

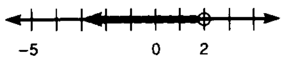
5. $[4,+\infty)$ 

7. $(-\infty,-1]$ 

9. $\{x|x > 9\}$ or $(9,+\infty)$ **11.** $\{x|x \geq 12\}$ or $[12,+\infty)$ **13.** $\{x|x \geq -5\}$ or $[-5,+\infty)$ **15.** $\left\{x\middle|x < \frac{3}{2}\right\}$ or $\left(-\infty,\frac{3}{2}\right)$ **17.** $\{x|x > 1\}$ or $(1,+\infty)$ **19.** $\left\{x\middle|x \geq -\frac{27}{2}\right\}$ or $\left[\frac{-27}{2},+\infty\right)$ **21.** $\left\{x\middle|x \geq \frac{7}{6}\right\}$ or $\left[\frac{7}{6},+\infty\right)$ **23.** $\left\{x\middle|x \geq \frac{1}{4}\right\}$ or $\left[\frac{1}{4},+\infty\right)$ **25.** $\{x|x < 3\}$ or $(-\infty,3)$ **27.** $\{x|x \geq -1\}$ or $[-1,+\infty)$ **29.** $\{x|x \geq 6\}$ or $[6,+\infty)$ **31.** $\{x|x \geq -14\}$ or $[-14,+\infty)$ **33.** $\{x|x \leq -1\}$ or $(-\infty,-1]$ **35.** $\left\{x\middle|\frac{2}{3} < x < 3\right\}$ or $\left(\frac{2}{3},3\right)$ **37.** $\left\{x\middle|\frac{2}{7} \leq x \leq \frac{8}{7}\right\}$ or $\left[\frac{2}{7},\frac{8}{7}\right]$ **39.** $\left\{x\middle|\frac{-10}{3} \leq x \leq 0\right\}$ or $\left[\frac{-10}{3},0\right]$ **41.** $\{x|-1 < x < 1\}$ or $(-1,1)$ **43.** $\left\{x\middle|4 \leq x \leq \frac{13}{2}\right\}$ or $\left[4,\frac{13}{2}\right]$ **45.** $\left\{x\middle|2 \leq x \leq \frac{7}{2}\right\}$ or $\left[2,\frac{7}{2}\right]$

47. $\left\{x \middle| \frac{-2}{3} \le x \le \frac{2}{3}\right\}$ or $\left[\frac{-2}{3}, \frac{2}{3}\right]$
49. (x = student's score), $x \ge 90$
51. (t = temperature), $t \le 42$ 53. (c = number of cars), $c \ge 10$
55. (m = number of minutes), $96 \le m \le 384$ 57. $\frac{3}{2}c \le P \le 3c$
59. $5x - 6 < 17$, $\left\{x \middle| x < \frac{23}{5}\right\}$ or $\left(-\infty, \frac{23}{5}\right)$
61. $\frac{1}{2}x + 16 > 24$, $\{x|x > 16\}$ or $(16, +\infty)$
63. $19 - 2x \le 8$, $\left\{x \middle| x \ge \frac{11}{2}\right\}$ or $\left[\frac{11}{2}, +\infty\right)$
65. $\frac{66 + 71 + 84 + x}{4} \ge 75$, minimum score is 79, $\{x|x \ge 79\}$ or $[79, +\infty)$ 67. $12 < 3x + 2 < 23$, $\left\{x \middle| \frac{10}{3} < x < 7\right\}$ or $\left(\frac{10}{3}, 7\right)$
69. $16 < 4s < 84$, $\{s|4 < s < 21\}$ or $(4,21)$

Solutions to trial exercise problems

6. $(-\infty, 2)$ or 

We use a hollow circle or a parenthesis at 2 to denote that we do not contain the endpoint (strict inequality).

13. $-4x \le 20$
$$\frac{-4x}{-4} \ge \frac{20}{-4}$$
$$x \ge -5$$
$\{x|x \ge -5\}$ or $[-5, +\infty)$

20. $4 - 2(3x + 1) > 8x - 12$
$$4 - 6x - 2 > 8x - 12$$
$$2 - 6x > 8x - 12$$
$$2 > 14x - 12$$
$$14 > 14x$$
$$1 > x$$
$\{x|x < 1\}$ or $(-\infty, 1)$

40. $2 \le 1 - x \le 6$
$$1 \le -x \le 5$$
$$\frac{1}{-1} \ge \frac{-x}{-1} \ge \frac{5}{-1}$$
$$-1 \ge x \ge -5$$
$\{x|-5 \le x \le -1\}$ or $[-5, -1]$

54. If x = the temperature, then $18 \le x \le 41$.

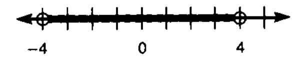
64. If x = the score on the fourth quiz, then
$$\frac{7 + 10 + 8 + x}{4} \ge 8$$
$$\frac{25 + x}{4} \ge 8$$
$$25 + x \ge 32$$
$$x \ge 7.$$
Therefore she must score 7 or more on the fourth quiz to have an average of 8 or more.

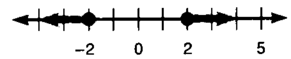
Review exercises

1. 21 2. 8 3. -2 4. $2y$ 5. $a + 6$
6. let x = the number; $\frac{1}{4}x$ 7. let x = the number; $x - 12$
8. let x = the number; $x + 7$

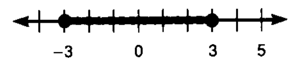
Exercise 2–6

Answers to odd-numbered problems

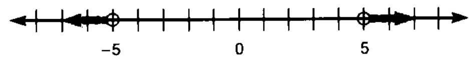
1. false 3. true 5. true 7. false
9. $\{x|-4 < x < 4\}$, $(-4,4)$ 
11. $\{x|x \le -2$ or $x \ge 2\}$, $(-\infty, -2] \cup [2, +\infty)$



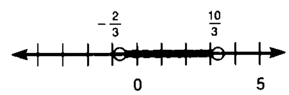
13. $\{x|-3 \le x \le 3\}$, $[-3,3]$



15. $\{x|x < -5$ or $x > 5\}$, $(-\infty, -5) \cup (5, +\infty)$



17. $\left\{x \middle| -\frac{2}{3} < x < \frac{10}{3}\right\}$, $\left(-\frac{2}{3}, \frac{10}{3}\right)$



19. $\left\{x \middle| x \le -\frac{4}{5} \text{ or } x \ge 2\right\}$, $\left(-\infty, -\frac{4}{5}\right] \cup [2, +\infty)$
21. $\left\{x \middle| -3 \le x \le \frac{17}{3}\right\}$, $\left[-3, \frac{17}{3}\right]$
23. $\left\{x \middle| \frac{-19}{5} < x < 1\right\}$, $\left(\frac{-19}{5}, 1\right)$
25. $\left\{x \middle| -3 \le x \le \frac{17}{3}\right\}$, $\left[-3, \frac{17}{3}\right]$
27. $\{x|-5 < x < 10\}$, $(-5,10)$ 29. $\emptyset$ 31. all real numbers, $\{x|x \in R\}$, $(-\infty, +\infty)$ 33. $\left\{x \middle| \frac{2}{7} < x < \frac{6}{7}\right\}$, $\left(\frac{2}{7}, \frac{6}{7}\right)$
35. $\left\{x \middle| x \le -2 \text{ or } x \ge \frac{8}{3}\right\}$, $(-\infty, -2] \cup \left[\frac{8}{3}, +\infty\right)$
37. $\emptyset$ 39. all real numbers, $\{x|x \in R\}$, $(-\infty, +\infty)$
41. $\left\{x \middle| x < \frac{1}{5} \text{ or } x > 1\right\}$, $\left(-\infty, \frac{1}{5}\right) \cup (1, +\infty)$
43. $\{x|x < 1.475$ or $x > 6.2\}$, $(-\infty, 1.475) \cup (6.2, +\infty)$
45. $\{x|-0.5 \le x \le 2\}$, $[-0.5, 2]$ 47. $\{x|-1 < x < 7\}$, $(-1,7)$
49. let x = the number; $|x| = 12$; $\{-12, 12\}$
51. let x = the number; $\left|\frac{1}{2}x\right| = 7$; $\{-14, 14\}$
53. let x = the number; $|4x + 3| = 19$; $\left\{-\frac{11}{2}, 4\right\}$
55. let x = the number; $|x| \le 6$, $\{x|-6 \le x \le 6\}$, $[-6,6]$
57. let x = the number; $|2x| < 14$, $\{x|-7 < x < 7\}$, $(-7,7)$
59. let x = the number; $|2x + 5| > 15$, $\{x|x < -10$ or $x > 5\}$, $(-\infty, -10) \cup (5, +\infty)$ 61. let x = the number; $\left|\frac{1}{4}x - 8\right| < 12$; $\{x|-16 < x < 80\}$, $(-16, 80)$

Solutions to trial exercise problems

24. $|1 - 2x| \le 5$

$$-5 \le 1 - 2x \le 5$$
$$-6 \le -2x \le 4$$
$$\frac{-6}{-2} \ge \frac{-2x}{-2} \ge \frac{4}{-2}$$
$$3 \ge x \ge -2$$

$\{x|-2 \le x \le 3\}$, $[-2,3]$

28. $|4x - 9| < 0$

$\emptyset$. The absolute value cannot be less than zero.

58. Let x = the number, then $|3x - 4| \ge 11$.

$3x - 4 \ge 11$ or $3x - 4 \le -11$

$3x \ge 15$ $\quad$ $3x \le -7$

$x \ge 5$ $\quad$ $x \le -\dfrac{7}{3}$

$\left\{x|x \le -\frac{7}{3} \text{ or } x \ge 5\right\}$, $\left(-\infty,-\frac{7}{3}\right] \cup [5,+\infty)$

Review exercises

1. -16 **2.** 16 **3.** -16 **4.** 16 **5.** x^5 **6.** x^3 **7.** x^2
8. xy

Chapter 2 review

1. $\{8\}$ **2.** $\{6\}$ **3.** $\{28\}$ **4.** $\{4\}$ **5.** $\left\{\frac{15}{7}\right\}$ **6.** $\left\{\frac{10}{3}\right\}$ **7.** $\left\{\frac{5}{3}\right\}$
8. $\left\{-\frac{13}{5}\right\}$ **9.** $\{9\}$ **10.** $w = \dfrac{v}{\ell h}$ **11.** $t = \dfrac{v - k}{g}$
12. $d = \dfrac{D - R}{q}$ **13.** $b = \dfrac{ar^2 - v}{r^2}$ **14.** $v = \dfrac{2s + gt^2}{2t}$
15. $n = \dfrac{\ell - a + d}{d}$ **16.** $x = \dfrac{3y}{2}$ **17.** 12 **18.** 36
19. $\frac{33}{5},\frac{99}{5},\frac{3}{5}$ **20.** 11 feet, 30 feet **21.** \$15,000 at 10%; \$9,000 at 8% **22.** 40 cl of 42% solution, 60 cl of 12% solution
23. $\{-15,15\}$ **24.** $\left\{-\frac{17}{3},\frac{7}{3}\right\}$ **25.** $\left\{-\frac{3}{2},\frac{17}{2}\right\}$ **26.** $\{0,3\}$
27. $\left\{-7,-\frac{1}{5}\right\}$ **28.** $\left\{-\frac{11}{2},-\frac{1}{6}\right\}$ **29.** $\{x|x \le 6\}$, $(-\infty,6]$
30. $\{x|x > 16\}$, $(16,+\infty)$ **31.** $\left\{x|x > -\frac{9}{2}\right\}$, $\left(-\frac{9}{2},+\infty\right)$
32. $\left\{x|x \le \frac{9}{8}\right\}$, $\left(-\infty,\frac{9}{8}\right]$ **33.** $\left\{x|x > -\frac{3}{2}\right\}$, $\left(-\frac{3}{2},+\infty\right)$
34. $\left\{x|x \le \frac{25}{3}\right\}$, $\left(-\infty,\frac{25}{3}\right]$ **35.** $\left\{x|-\frac{7}{2} < x < 1\right\}$, $\left(-\frac{7}{2},1\right)$
36. $\left\{x|-\frac{4}{5} \le x \le 0\right\}$, $\left[-\frac{4}{5},0\right]$ **37.** $\{x|-5 < x < -2\}$, $(-5,-2)$ **38.** $\left\{x\middle|\frac{2}{3} < x \le \frac{10}{3}\right\}$, $\left(\frac{2}{3},\frac{10}{3}\right]$
39. $\{x|-1 \le x < 0\}$, $[-1,0)$ **40.** let x = the number; $4x - 5 \ge 19$, $\{x|x \ge 6\}$, $[6,+\infty)$ **41.** let x = the number; $22 < 3x + 7 < 34$, $\{x|5 < x < 9\}$, $(5,9)$
42. $\{x|x \le -10 \text{ or } x \ge 10\}$, $(-\infty,-10] \cup [10,+\infty)$
43. $\left\{x|-\frac{11}{2} < x < \frac{1}{2}\right\}$, $\left(-\frac{11}{2},\frac{1}{2}\right)$
44. $\left\{x|-\frac{6}{5} \le x \le \frac{8}{5}\right\}$, $\left[-\frac{6}{5},\frac{8}{5}\right]$
45. $\left\{x|x > \frac{1}{2} \text{ or } x < -4\right\}$, $(-\infty,-4) \cup \left(\frac{1}{2},+\infty\right)$
46. $\left\{x|x \le -\frac{4}{3} \text{ or } x \ge 2\right\}$, $\left(-\infty,-\frac{4}{3}\right] \cup [2,+\infty)$
47. $\left\{x|-\frac{9}{4} < x < \frac{15}{4}\right\}$, $\left(-\frac{9}{4},\frac{15}{4}\right)$
48. all real numbers, $\{x|x \in R\}$,$(-\infty,+\infty)$
49. $\left\{x|-\frac{3}{2} < x < \frac{5}{2}\right\}$, $\left(-\frac{3}{2},\frac{5}{2}\right)$
50. $\left\{x|x \le -\frac{9}{2} \text{ or } x \ge 2\right\}$, $\left(-\infty,-\frac{9}{2}\right] \cup [2,+\infty)$ **51.** $\emptyset$

Chapter 2 cumulative test

1. false **2.** false **3.** true **4.** true **5.** true **6.** true
7. true **8.** 24 **9.** -1 **10.** 12 **11.** 0 **12.** 37 **13.** -6
14. -49 **15.** commutative property of multiplication
16. commutative property of multiplication **17.** $\{1,2,3,4,5\}$
18. $\{4,6,8,9,10,11\}$ **19.** $\{10,12\}$ **20.** $\left\{\frac{7}{4}\right\}$
21. $P = -\dfrac{M}{\ell - x}$ or $\dfrac{M}{x - \ell}$
22. $P_2 = \dfrac{P + nP_1 + c}{n}$ **23.** $\{x|x \ge -5\}$
24. $\left\{x|x \le -\frac{11}{4} \text{ or } x \ge -\frac{1}{4}\right\}$ **25.** $\left\{\frac{21}{17}\right\}$ **26.** $\left\{x|x \ge \frac{15}{4}\right\}$
27. $\{-1,1\}$ **28.** $\left\{\frac{16}{7}\right\}$ **29.** $\emptyset$ **30.** $\left\{x|-1 \le x \le \frac{11}{3}\right\}$
31. 16,8,48 **32.** \$26,000 at 11%; \$14,000 at 8%

Chapter 3

Exercise 3-1

Answers to odd-numbered problems

1. $(-2)^4$, -2 base, 4 exponent **3.** x^5, x base, 5 exponent
5. $(2x)^4$, $2x$ base, 4 exponent **7.** $(x^2 + 3y)^3$, $x^2 + 3y$ base, 3 exponent **9.** -2^2, 2 base, 2 exponent **11.** a^9 **13.** y^3
15. $(-2)^6 = 64$ **17.** $(-2)^4 = 16$ **19.** -36 **21.** x^8
23. x^7y^5 **25.** $6a^3b^2$ **27.** $6x^5$ **29.** $24x^3y^9$ **31.** $2^6 = 64$
33. -729 **35.** 64 **37.** a^{12} **39.** $x^{10}y^5z^{15}$ **41.** $49s^8t^4$
43. $x^{36}y^{48}z^{32}$ **45.** a^9b^{17} **47.** $x^{13}y^{24}$ **49.** $-x^{10}y^{18}$
51. $-75x^{10}y^{11}$ **53.** $17x^9$ **55.** $-76a^{13}$ **57.** $9x^{10} - 8x^8$
59. $24a^{13} + 18a^{11}$ **61.** $3x^{17} + 96x^{14}$ **63.** a^{9b} **65.** a^{9b}
67. a^{5b+3} **69.** x^{3y+5} **71.** x^{15y^2} **73.** \$5,634.13
75. 2.5 grams

Solutions to trial exercise problems

18. $(-2)(-2^2) = (-2)(-4) = 8$ **32.** $(-2^2)^3 = (-4)^3 = -64$
44. $(3x^2y)^2(2xy^3) = 9x^4y^2 \cdot 2xy^3 = 18x^5y^5$
52. $(2a^2)^2a^3 + (3a)^3a^4 = 4a^4a^3 + 27a^3a^4 = 4a^7 + 27a^7 = 31a^7$
57. $(3x^5)^2 - (2x^2)^3x^2 = 9x^{10} - 8x^6x^2 = 9x^{10} - 8x^8$. The subtraction cannot be performed because we do not have like terms.
62. $x^{5n} \cdot x^{4n} = x^{5n+4n} = x^{9n}$
66. $x^{2n+1} \cdot x^{n+4} = x^{(2n+1)+(n+4)} = x^{2n+1+n+4} = x^{3n+5}$
70. $(a^{3n})^{4n} = a^{3n \cdot 4n} = a^{12n^2}$

Review exercises

1. -24 **2.** -9 **3.** 0 **4.** 25 **5.** $9ab$ **6.** $a^2 - 2a - 15$
7. $x^2 - 9$ **8.** $x^2 + 2y^2$

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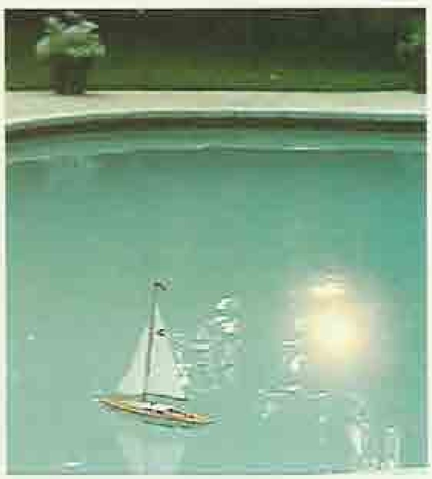

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